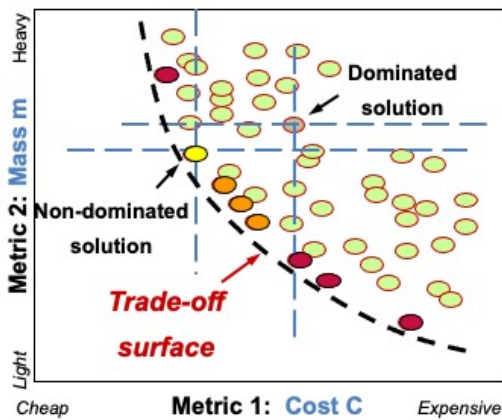




Unit 8.

Objectives in conflict: trade off methods and penalty functions

Mike Ashby

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This lecture unit is part of a set created by Mike Ashby to help introduce students to materials, processes and rational selection.

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The 25 PowerPoint Lecture Units For 2015

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Learning objectives for Lecture Unit 8

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Learning Objectives/Intended Learning Outcomes

Knowledge and Understanding	Knowledge on graphical trade-off methods and penalty functions
Skills and Abilities	Ability to select systematically when design objectives conflict
Values and Attitudes	Appreciation of the value of compromise in engineering design

Resources

- Text: "**Materials Selection in Mechanical Design**", 4th Edition by M.F. Ashby, Butterworth Heinemann, Oxford, 2011. Chapters 7-8
- Text: "**Materials and the Environment**", 2nd Edition by M.F. Ashby, Butterworth-Heinemann, Oxford 2012, UK. Chapters 9-10



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Learning Objectives

These Intended Learning Outcomes are based on a taxonomy of *knowledge and understanding* as the basis, *skills and abilities* as necessary for the practical use of knowledge and understanding, followed by acquired *values and attitudes* enabling assessments and responsible use of these abilities.

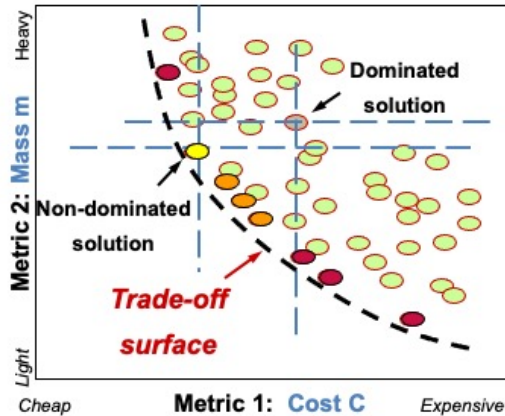
Combined with a suitable assessment, they should be helpful in the context of accreditations, such as ABET, or for the CDIO Syllabus.

The Texts listed are from books authored or co-authored by Mike Ashby.



Outline of Lecture Unit 8

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- Almost always **2 + objectives** – they conflict
- **Trade-off methods**
- **Penalty functions** and exchange constants

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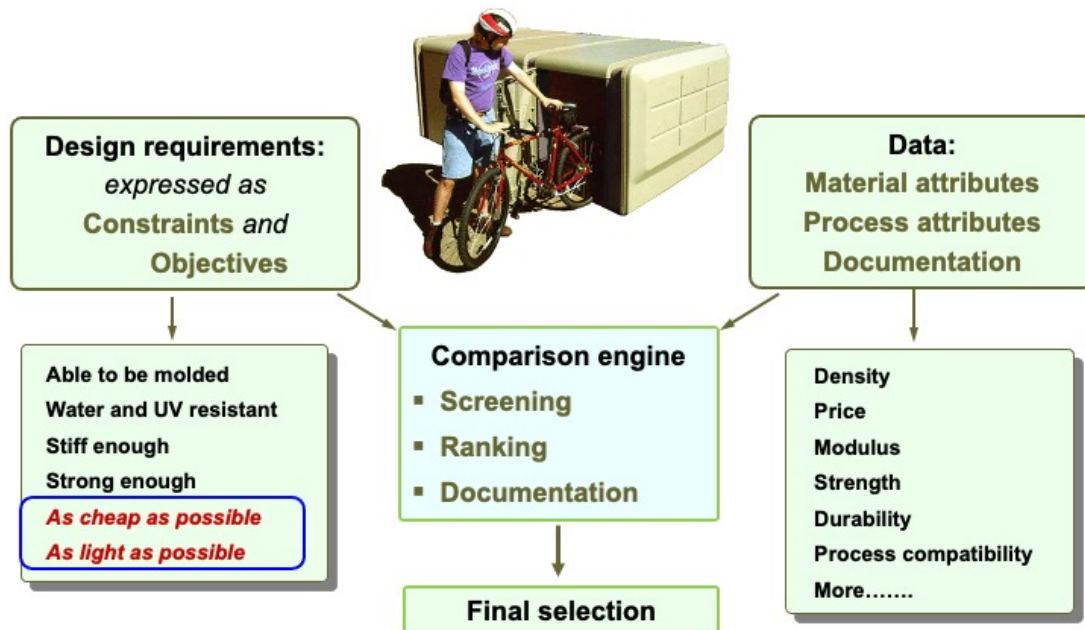
Outline

Real-life decision-making frequently requires that a compromise be reached between **conflicting objectives**. Some are only too familiar: the compromises required to strike a balance between the performance and the cost of a car for example, or between health and the pleasure of eating rich foods, or between wealth and quality of life. Conflict arises because the choice that optimizes one objective will not, in general, do the same for the others; then the best choice is a compromise, optimizing none but pushing all as close to their optima as their interdependence allows.



The selection strategy: materials

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A reminder: the selection strategy

This frame illustrates the decision-making strategy applied to the selection of a material.

- The **Design requirements** (upper left) are expressed as constraints that the material must meet and the objectives, defined in a moment, that are chosen as measures of the excellence of choice.
- The **Data** (upper right) takes the form of a database of the attributes of the materials and processes that are possible candidates for the design
- The **comparison engine** applies the constraints, eliminating materials that cannot meet the requirements, and then ranks the survivors, using the objectives, to create a short list. The final choice is made by exploring documentation of the top-ranked candidates.



Multiple constraints and objectives

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Design requirements set **constraints** – criteria for screening
objectives – criteria for optimising

Typical constraints

The material must be

- Electrically conducting
- Optically transparent.....

And meet target values of

- Stiffness
- Strength.....

And be able to be

- Die cast
- Welded

Dealing with multiple constraints is straightforward

Typical objectives

Minimize

- **Mass m** (*satellite components*)
- **Volume** (*mobile phones*)
- **Energy consumption** (*fridges*)
- **Carbon footprint** (*cars*)
- **Embodied energy** (*materials*)
- **Cost C** (*everything*)

Dealing with multiple objectives needs **trade-off methods**

Take, as example, simultaneously minimizing **mass m** and **cost C**

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Multiple constraints and objectives

This frame lists, on the left, typical constraints that a material must meet. Dealing with **multiple constraints** is straightforward – just apply them using Limit, Graph and Tree stages. On the right is a list of typical objectives. Dealing with multiple objectives is more complicated.

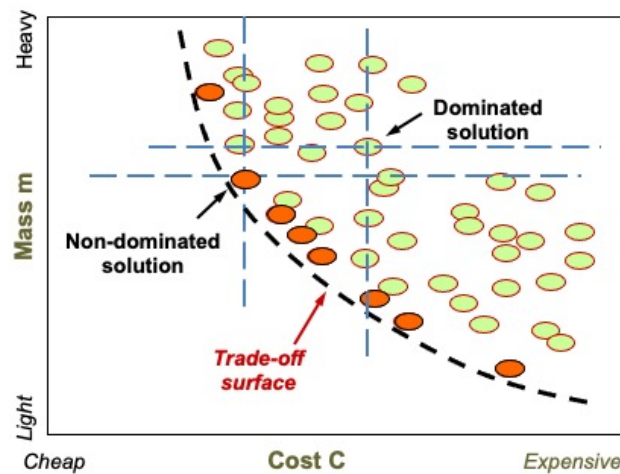
An **objective**, it will be remembered from Units 6 and 7, defines a performance metric. If the objective is to *minimize mass*, then the mass becomes the metric of “goodness” or “badness” of a given choice: the lightest solution that meets all the constraints of the problem is the best choice. If the objective is to *minimize cost*, then the cheapest solution that meets all constraints is the best choice. The metric allows solutions to be ranked. This frame lists common design objectives; there are, of course, many more. It is rare that a design has only one objective. And when there are two a conflict arises: the choice that minimizes one metric – mass say – does not generally minimize the other – cost, for example. Then a compromise must be sought. To reach it we need some simple ideas drawn from the field of **multi-objective optimization**.



Multi-objective optimization: *the words*

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- **“Solution”**: a candidate that meets the constraints, but not necessarily optimum by either objective
- Plot solutions.
(**Convention**: express objectives to be **minimized**)
- **“Dominated solution”**: one that is definitely non-optimal
- **“Non-dominated solution”**: one that is optimal by one metric (but not usually by both)
- **“Trade-off surface”**: the surface on which the non-dominated solutions lie (Pareto Front)



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The terminology of multi-objective optimization

Multi-objective optimization is a technique for reaching a compromise between conflicting objectives. It lends itself to visual presentation in a way that fits well with methods developed here thus far. This frame explains the words. They are illustrated by the diagram on the right in which we have specialized a problem to a trade-off between the mass of a component and its cost.

The first bullet point on the frame defines a *solution*: a choice of material to make a component that meets all the necessary constraints and is thus a candidate for the design, although not perhaps the best one. The little circles each represent a solution; each describes the mass and cost of the component if made from a given material. The next two bullet points distinguish between a *dominated* solution (meaning that other solutions exist that are both lighter and cheaper) and a *non-dominated* solution (one that is lighter than all others that cost less and cheaper than all others that are lighter – thus there is no other solution that is both lighter and cheaper than it is). The lower envelope links non-dominated solution. It defines the **trade-off surface** or **Pareto front**. Solutions that lie on or near the trade-off surface are a better choice than those that do not.

We adopt the convention that each performance metric is defined in such a way that a **minimum** is sought for it. For mass and cost, that is exactly what we want. But if the metric were maximum speed v (a performance objective for a sports car, for instance) we must invert it and seek a minimum for $1/v$. With this convention the trade-off surface must have a negative slope everywhere, as that in the schematic does. A positive slope would link non-dominated solutions.

With this background we can examine strategies for finding the best compromise. There are three.



Finding a compromise: strategy 1

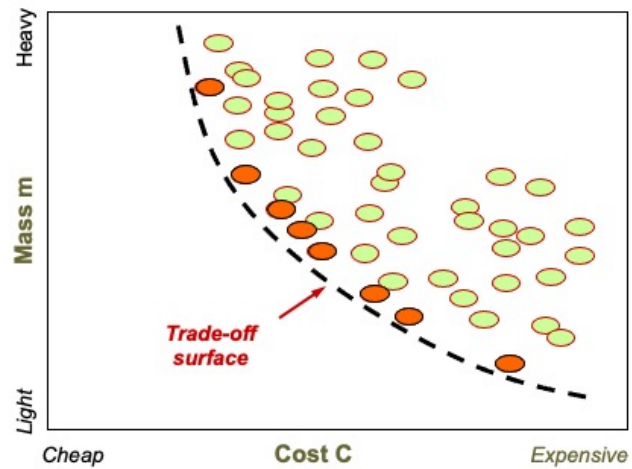
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- Make trade-off plot
- Sketch trade-off surface
- Use intuition to select a solution on the trade-off surface

▪ “Solutions” nearest the surface offer the best **compromise** between mass and cost

▪ 8 out of 50

- Choose from among these - depends on how highly you value light weight



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The simplest approach

The solutions on or near the trade-off surface, here colored red, offer a better compromise between mass and cost than those that do not. This immediately isolates a subset of the entire population of solutions, identifying these as the best candidates. It is a big step forward, but it still leaves us with a choice: which part of the trade-off surface is the best? The first strategy is to use **intuition** (experience, good judgment, common sense – call it what you like) for guidance, selecting materials from among the non-dominated (red-colored) set.



Finding a compromise: strategy 2

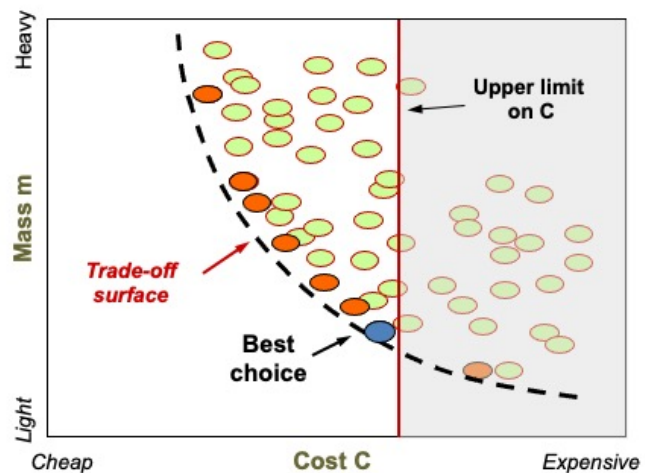
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▪ **Reformulate** all but one of the objectives as constraints, setting an upper limit for it

OK if budget limit

• BUT....cheating

Cost is treated as **constraint**, not **objective**.



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Strategy 2: treating objectives as constraints

The second tactic is to **impose an upper limit on one of the metrics** – cost, say – allowing any choice that is less than this limit. Then it's easy. Choose the solution on the trade-off surface that comes just under the limit. If you were choosing a car and wanted the fastest but had a definite budget limit, then this is the way to do it. But it is an extreme sort of optimization: cost has been treated as a constraint, not an objective. Strategies 1 and 2 help with all trade-off problems in material selection, but they rely to some extent on judgment. A more systematic method is possible – it comes next.



Finding a compromise: strategy 3

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Define locally-linear
Penalty function Z

$$Z = \alpha m + C$$

Seek solution with smallest Z

Make trade-off plot

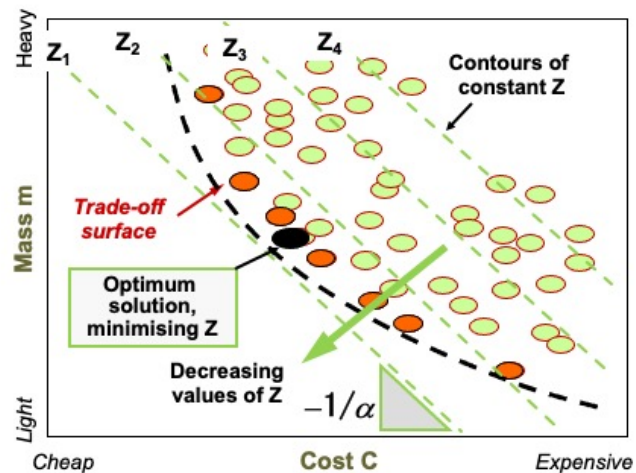
Plot on it contours of Z

$$m = -\frac{1}{\alpha}C + \frac{1}{\alpha}Z$$

Lines of Z have slope $-1/\alpha$
(needs linear scales)

Read off solution with lowest Z

But what is α ?



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Strategy 3 – define a Penalty function

There is a more formal, systematic, way to find the best compromise, although it is not always practical to use it. We define a **locally-linear penalty function** Z (a global objective) combining the two metrics mass, m , and cost, C :

$$Z = \alpha m + C$$

and seek the solution that minimizes Z (assuming we have a value for the constant α). That can be done by simply calculating Z for each solution and ranking the solutions by this value, or it can be done graphically in the way shown on this frame. Rearranging the equation for Z gives

$$m = (-1/\alpha)C + (1/\alpha)Z$$

This equation describes a family of parallel lines with slope $-1/\alpha$, each line corresponding to a value of Z , as shown. The best choices lie near the point at which one of these lines is tangent to the trade-off surface, since this minimizes Z .

To plot the contours we need a value for α . That depends on the application. We analyse this in a moment, but first some examples.



Materials for transport systems

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Steel



Steel / Alu



Alu / (composite)



Alu / Ti / composites



Composites

Choice of material depends on system

- **Mass**, in transport systems, means **fuel** \$
- **Life cost** = Initial cost, **C** + Fuel cost over life, scaling with mass **m** kg
- **Penalty function** $Z = C + \alpha m$ \$/kg
- Must first establish **exchange constant**, α

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The importance of the exchange constant

Now a materials selection example: structural materials for transport systems. Five systems are shown along the top of the frame, with the dominant choice of structural material listed beneath ranging from steel on the left, aluminum in the middle and advanced composites on the right.

The goal is to minimize life cost. It is the sum of the initial cost and the cost over life, dominated by fuel cost (fuel consumption scales with mass). The two are combined in a penalty function Z . The quantity α is called an **exchange constant** (or "**parameter influence coefficient**") because it converts the units of one metric – mass – into the other – cost (like the currency exchange rate that converts one currency into another). It measures the value of a unit change of the performance metric m : it is the value associated with unit reduction in mass, and so has the units €/kg or \$/kg.

Unit 6 developed indices for mass and cost for components loaded in bending (the commonest mode of loading) – they are listed on the frame, and combined to give the penalty function. To evaluate it we need a value for the exchange constant, α – the cost-penalty of mass.



The exchange constant α for transport

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Exchange constants for transport systems

Transport system	α (\$ per kg)
Family car	3 to 6
Truck	5 to 20
Civil aircraft	100 to 500
Military hardware	500 to 2000
Space vehicle	3000 to 10,000

How to get values of α ?

- Full life costing: fuel saving
- Value of extra payload
- Extrapolation of historic data



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Values of exchange constants for transport systems

The table lists approximate **values for the exchange constant α for transport systems**, based on the economic benefit of a reduction in structural mass of 1kg, all other things remaining the same. For the family car it is calculated from the fuel saving over a life of 100,000 km. For the truck, aircraft and spacecraft it is calculated from the value of an additional 1kg of payload over the operating life.

The values vary widely. The value of weight saving in a car is small; that is one reason that it is difficult to replace steel with a lighter metal in cars – the weight (and thus fuel) saving does not compensate for the higher cost of the material. But in space it is different: here, because launch costs per kg are so enormous, the saving of mass is valued highly, making it economic to use even very expensive materials if they save weight.

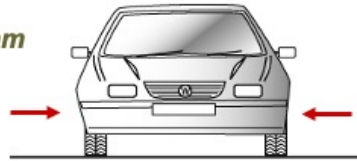
These values for exchange constants are based on engineering criteria. Sometimes, however, value is set in other ways. The *perceived value* of a product is an important factor in marketing. It is measured – or estimated – by market surveys, questionnaires and the like.



Materials for auto bumpers

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Beam



Function

Absorb impact, transmit load to energy-absorbing units or supports

Objectives

Minimize mass and material cost

Criteria

Mass m per unit
bending strength

Cost C per unit
bending strength

Index: minimize

$$m = \frac{\rho}{\sigma_y^{2/3}}$$

$$C = \frac{C_m \rho}{\sigma_y^{2/3}}$$

C_m = Material cost / kg
 ρ = Density, kg/m³
 σ_y = Yield strength, MPa
 α = exchange constant, \$/kg

Penalty function

$$Z = C + \alpha m = \frac{\rho}{\sigma_y^{2/3}} (C_m + \alpha)$$

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An example: balancing cost and mass for an auto component

Bumpers of road vehicles protect the vehicle and its passengers in the event of impact. The bumper is part of the vehicle; it adds to its weight and thus to its fuel consumption. We can now evaluate the penalty function derived earlier to select materials for different classes of vehicle. To do so we evaluate the penalty function, using the “Advanced” facility in CES EduPack to plot it.



Selection using the penalty function

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$$Z = \frac{\rho}{\sigma_y^{2/3}} (C_m + \alpha)$$

Use the
“**Advanced**”
facility to make
the penalty
function

(Density / (Yield strength^0.66))
*(Price + 10)

The value of the
exchange constant

+ - / * ^ ()

List of properties

- Density
- Price
- Tensile strength
- etc

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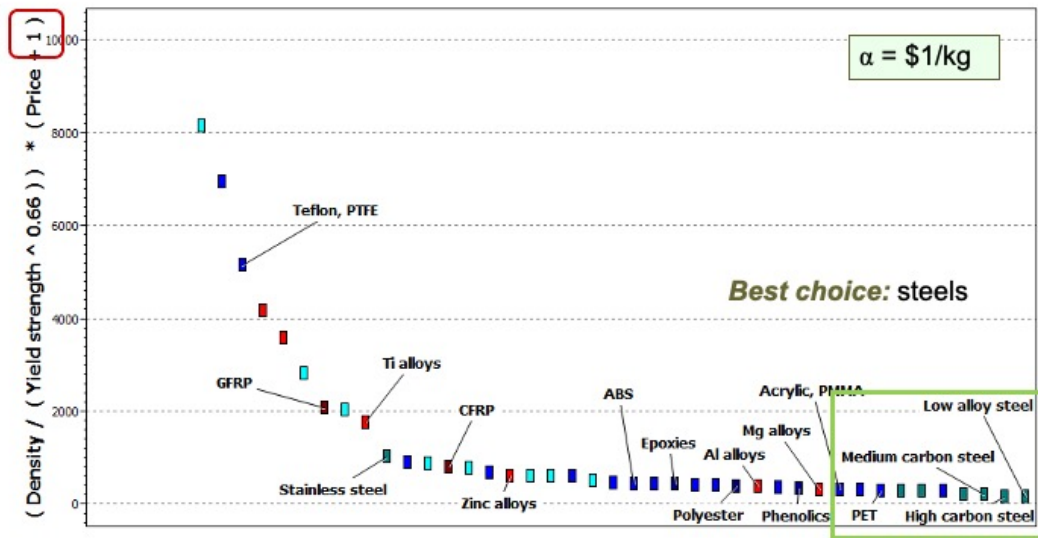
Using the function-generator to plot penalty functions

The **penalty function Z** is plotted as a bar chart in the way shown here. The best choice of material – the one that minimizes life-cost – is that with the lowest value of Z. The result depends on the value of the exchange constant.



Selection using the penalty function

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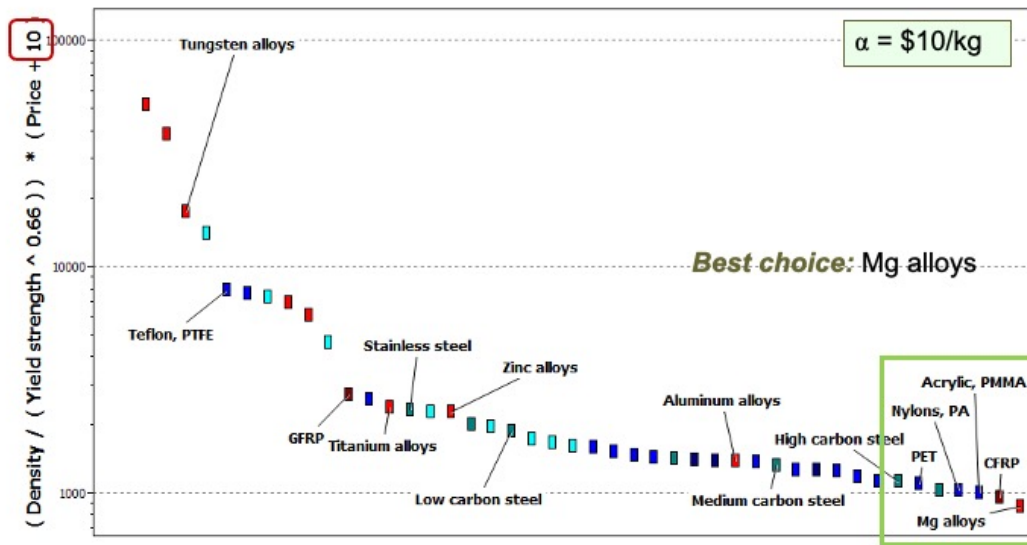
The penalty function with $\alpha = 1$

The **penalty function Z** is plotted as a bar chart in the way shown here. The best choice of material – the one that minimizes life-cost – is that with the lowest value of Z. The result depends on the value of the exchange constant.



Selection using the penalty function

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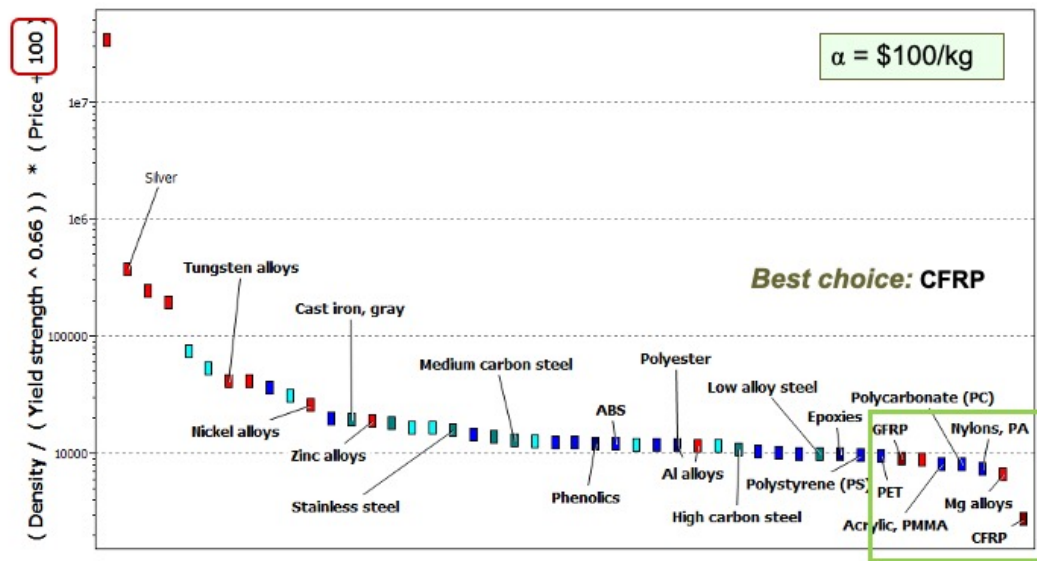
The penalty function with $\alpha = 10$

The **penalty function Z** is plotted as a bar chart in the way shown here. The best choice of material – the one that minimizes life-cost – is that with the lowest value of Z. The result depends on the value of the exchange constant.



Selection using the penalty function

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The penalty function with $\alpha = 100$

The **penalty function Z** is plotted as a bar chart in the way shown here. The best choice of material – the one that minimizes life-cost – is that with the lowest value of Z. The result depends on the value of the exchange constant.



So what?

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- Real design involves conflicting objectives – often **technical performance** vs. **economic performance** (cost).
- **Trade-off plots** reveal options
- If **exchange constant** is known – **penalty function** allows unambiguous choice

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Summary

This unit has introduced ways of dealing with **conflicting objectives** in materials selection. The key concept is that of the **trade-off plot** – it alone is often enough to identify good choices. If greater precision is required, the **penalty function** method provides it.



Lecture Unit series

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Finding and Displaying Information	
Unit 1	The materials of engineering
Unit 2	Material property charts: mapping the materials universe
Unit 3	The Elements database: properties, relationships and resources
Material Properties	
Unit 4	Manipulating properties: chemistry, microstructure, architecture
Unit 5	Designing new materials: filling the materials-property space
Selection	
Unit 6	Material selection: translation, screening, documentation
Unit 7	Ranking: material indices
Unit 8	Objectives in conflict: trade off methods and penalty functions
Unit 9	Material and shape: materials for efficient structures
Unit 10	Manufacturing processes: shaping, joining and surface treatment
Unit 11	Processes and cost modelling
Unit 12	Eco selection and the Eco Audit tool

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What is a Sustainable development? a materials perspective
Materials for low carbon power
Special Topics
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