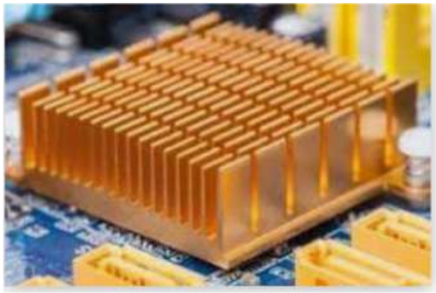
Selection of Material and Shape

A few examples on the shape – performance relationship…

How shape can be used to modify the ways in which materials behave?

Whis glass is used to consume a cold beverage?

Heat Sink???



Fins on the engine

Torsion Box?

Soundproof room

A few examples on the shape – performance relationship…

How shape can be used to modify the ways in which materials behave?

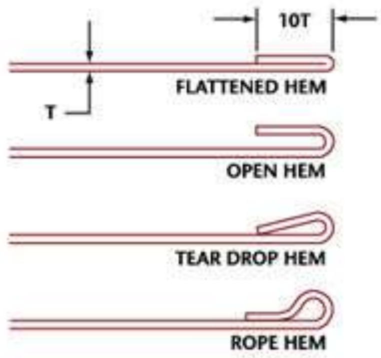
Metamaterials

Can a sheet of paper carry load?

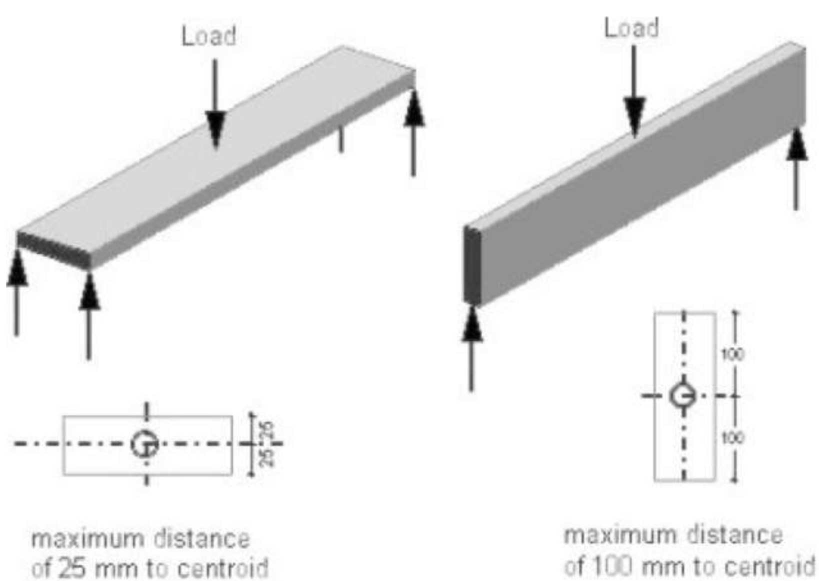
https://www.vectorstock.com/royalty-free-vector/white-sheet-of-paper-background-vector-3726748

Macrostructure – Efficiency

Designer vs. Materials Engineer



Does the elastic modulus change when a sheet of metal is folded?



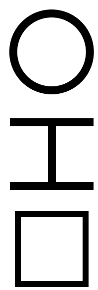
http://www.snobarcolorbar.com/technical-data/

Structural sections

When materials are loaded in bending, in torsion, or are used as slender columns, ***section shape*** becomes important

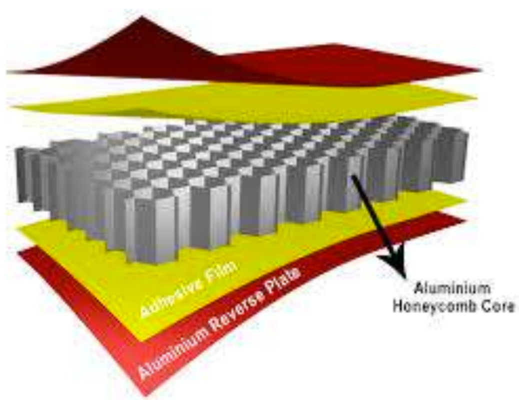
Shape = cross section formed to a

- Tube
- I-section
- Hollow box

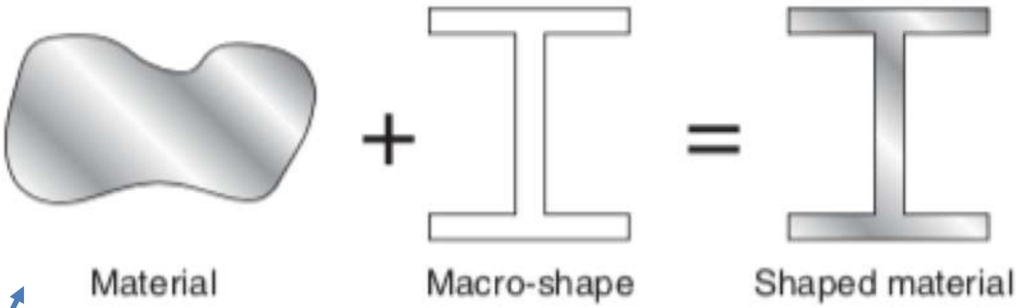


Simple shapes

Complex shapes (relatively higher efficiency)



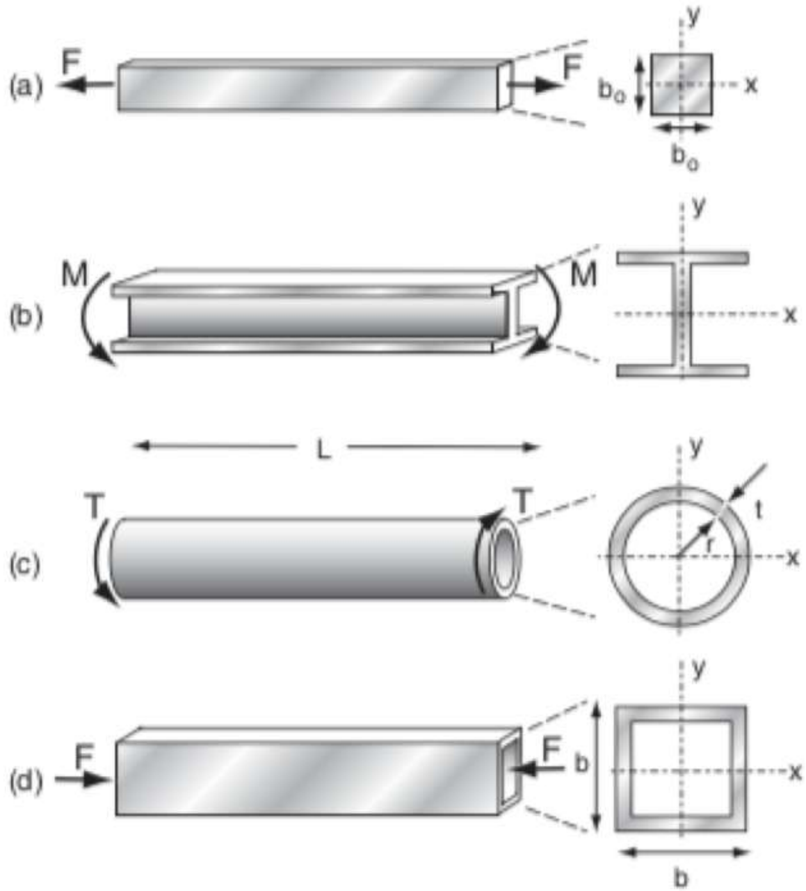
Shape Factors



Normalized!

Mechanical efficiency is obtained by combining material with macroscopic shape. The shape is characterized by a dimensionless shape factor, ϕ. The schematic is suggested by Parkhouse (1984).

Shape Factors



Tie

Beam

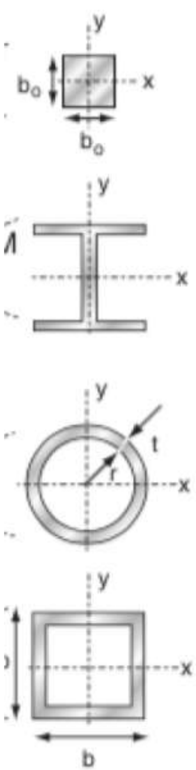
Shaft

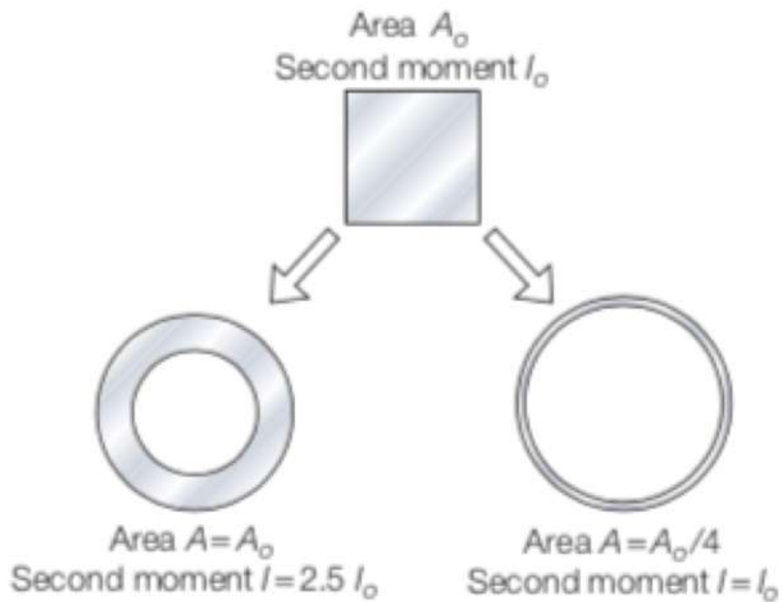
Colums

Common modes of loading and the section-shapes that are chosen to support them: (a) axial tension (b) bending (c) torsion and (d) axial compression, which can lead to buckling.

Shape Factors

Mass distribution and stiffness.

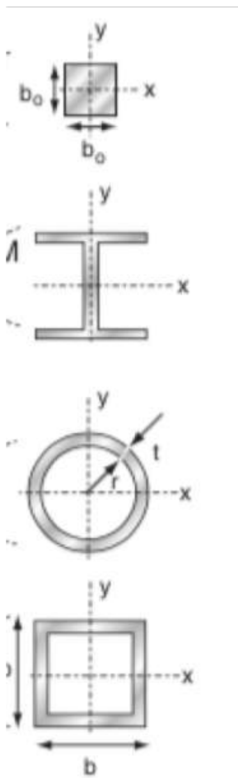


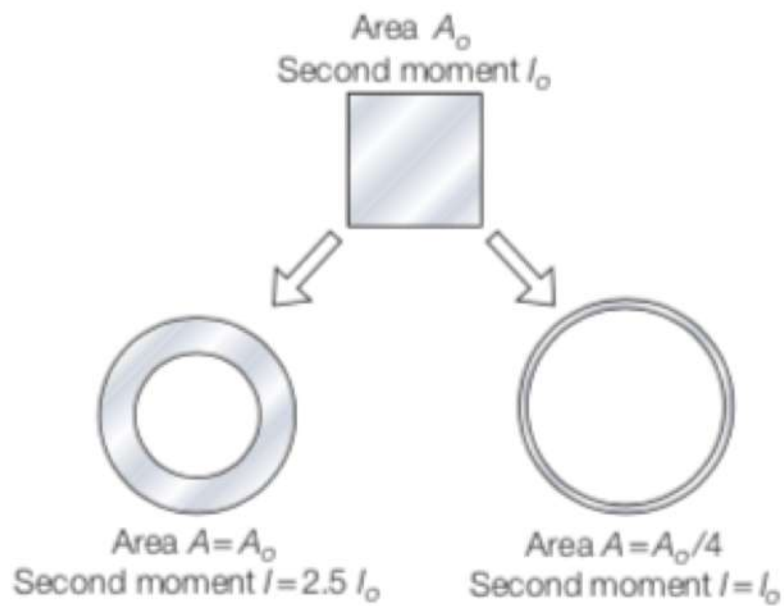


Shape Factors

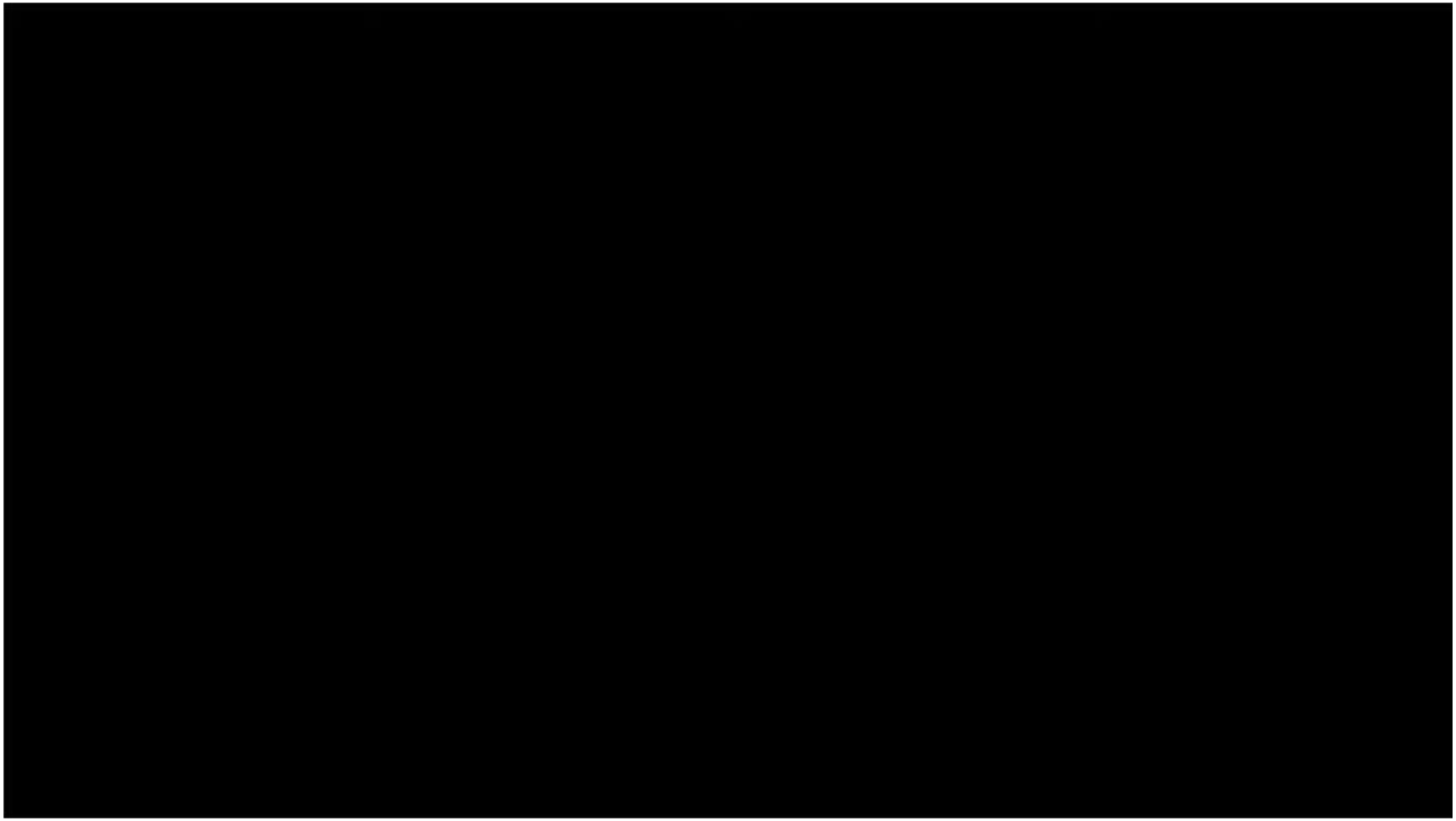
$S \propto EI$ / L^3

Mass distribution and stiffness.





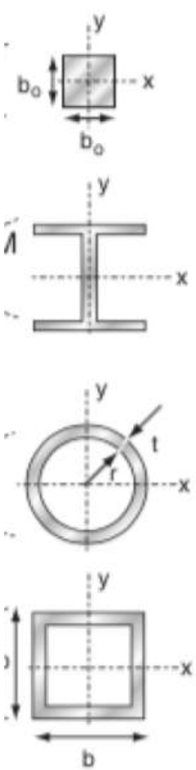
Shape Factors

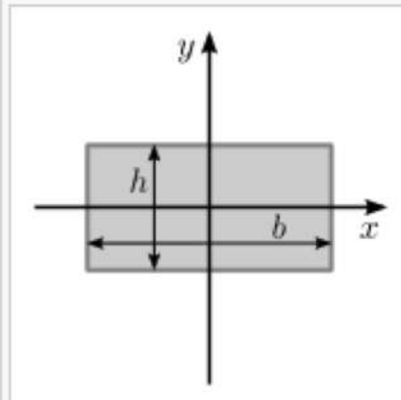


Shape Factors

$S \propto EI$ / L^3

Mass distribution and stiffness.





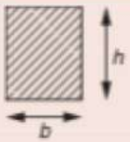
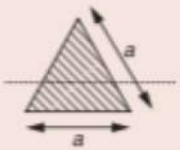
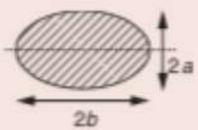
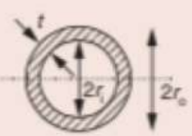
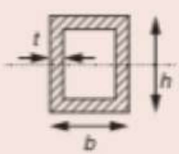
Rectangle with base *b* and height *h*

$$I_x = \iint_R y^2 \, dA = \int_{-\frac{b}{2}}^{\frac{b}{2}} \int_{-\frac{h}{2}}^{\frac{h}{2}} y^2 \, dy \, dx = \int_{-\frac{b}{2}}^{\frac{b}{2}} \frac{1}{3} \frac{h^3}{4} \, dx = \frac{bh^3}{12}$$

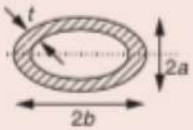
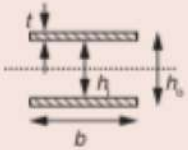
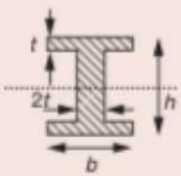
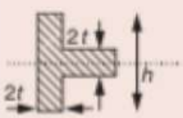
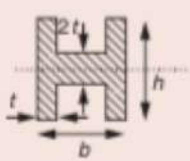
https://en.wikipedia.org/wiki/Second_moment_of_area

Elastic Bending of Beams

Moments of sections, and units

Section shape	Area A (m)	Moment I (m^4)	Moment K (m^4)	Moment Z (m^4)	Moment Q (m^4)	Moment Z_p (m^4)
	bh	$\frac{bh^3}{12}$	$\frac{bh^3}{3}\left(1-0.58\frac{b}{h}\right)$ $(h>b)$	$\frac{bh^2}{6}$	$\frac{b^2h^2}{(3h+1.8b)}$ $(h>b)$	$\frac{bh^2}{4}$
	$\frac{\sqrt{3}}{4}a^2$	$\frac{a^4}{32\sqrt{3}}$	$\frac{\sqrt{3}\,a^4}{80}$	$\frac{a^3}{32}$	$\frac{a^3}{20}$	$\frac{3a^3}{64}$
	πr^2	$\frac{\pi}{4}r^4$	$\frac{\pi}{2}r^4$	$\frac{\pi}{4}r^3$	$\frac{\pi}{2}r^3$	$\frac{\pi}{3}r^3$
	πab	$\frac{\pi}{4}a^3b$	$\frac{\pi a^3b^3}{(a^2+b^2)}$	$\frac{\pi}{4}a^2b$	$\frac{\pi}{2}a^2b$ $(a<b)$	$\frac{\pi}{3}a^2b$
	$\pi(r_o^2-r_i^2)$ $\approx 2\pi rt$	$\frac{\pi}{4}(r_o^4-r_i^4)$ $\approx \pi r^3t$	$\frac{\pi}{2}(r_o^4-r_i^4)$ $\approx 2\pi r^3t$	$\frac{\pi}{4r_o}(r_o^4-r_i^4)$ $\approx \pi r^2t$	$\frac{\pi}{2r_o}(r_o^4-r_i^4)$ $\approx 2\pi r^2t$	$\frac{\pi}{3}(r_o^3-r_i^3)$ $\approx \pi r^2t$
	$2t(h+b)$ $(h, b \gg t)$	$\frac{1}{6}h^3t\left(1+3\frac{b}{h}\right)$	$\frac{2tb^2h^2}{(h+b)}\left(1-\frac{t}{h}\right)^4$	$\frac{1}{3}h^2t\left(1+3\frac{b}{h}\right)$	$2tbh\left(1-\frac{t}{h}\right)^2$	$bht\left(1+\frac{h}{2b}\right)$

Elastic Bending of Beams

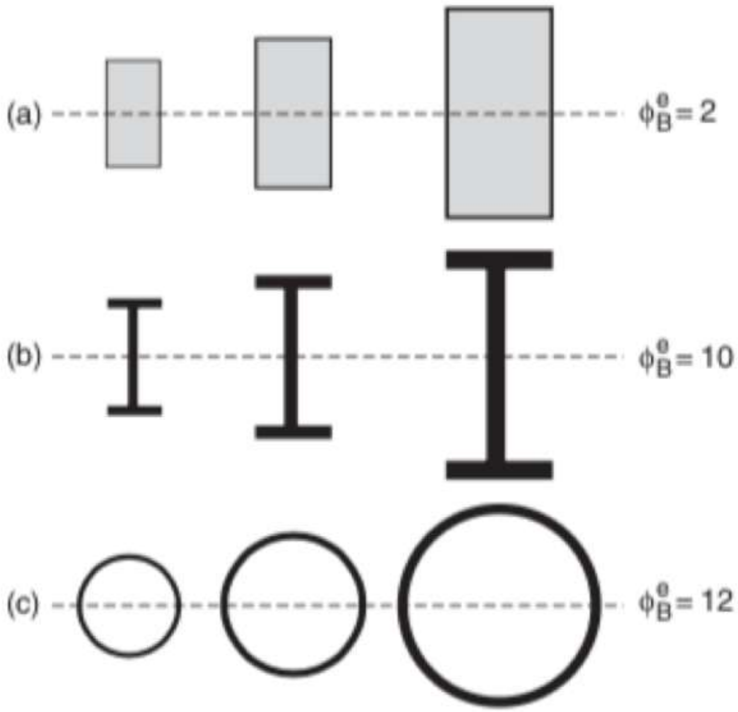
	$\pi(a+b)t$ $(a, b \gg t)$	$\frac{\pi}{4}a^3t\left(1+\frac{3b}{a}\right)$	$\frac{4\pi(ab)^{5/2}t}{(a^2+b^2)}$	$\frac{\pi}{4}a^2t\left(1+\frac{3b}{a}\right)$	$2\pi t(a^3b)^{1/2}$ $(b > a)$	$\pi abt\left(2+\frac{a}{b}\right)$
	$b(h_o - h_i)$ $\approx 2bt$ $(h, b \gg t)$	$\frac{b}{12}(h_o^3 - h_i^3)$ $\approx \frac{1}{2}bth_o^2$	—	$\frac{b}{6h_o}(h_o^3 - h_i^3)$ $\approx bth_o$	—	$\frac{b}{4}(h_o^2 - h_i^2)$ $\approx bth_o$
	$2t(h+b)$ $(h, b \gg t)$	$\frac{1}{6}h^3t\left(1+3\frac{b}{h}\right)$	$\frac{2}{3}bt^3\left(1+4\frac{h}{b}\right)$	$\frac{1}{3}h^2t\left(1+3\frac{b}{h}\right)$	$\frac{2}{3}bt^2\left(1+4\frac{h}{b}\right)$	$bht\left(1+\frac{h}{2b}\right)$
	$2t(h+b)$ $(h, b \gg t)$	$\frac{t}{6}(h^3+4bt^2)$	$\frac{t^3}{3}(8b+h)$	$\frac{t}{3h}(h^3+4bt^2)$	$\frac{t^2}{3}(8b+h)$	$\frac{th^2}{2}\left\{1+\frac{2t(b-2t)}{h^2}\right\}$
	$2t(h+b)$ $h, b \gg t)$	$\frac{t}{6}(h^3+4bt^2)$	$\frac{2}{3}ht^3\left(1+4\frac{b}{h}\right)$	$\frac{t}{3h}(h^3+4bt^2)$	$\frac{2}{3}ht^2\left(1+4\frac{b}{h}\right)$	$\frac{th^2}{2}\left\{1+\frac{2t(b-2t)}{h^2}\right\}$

Elastic Bending of Beams

$$I_o = \frac{b_o^4}{12} = \frac{A^2}{12}$$

$$\phi_B^e = \frac{S}{S_o} = \frac{EI}{EI_o} = \frac{12\,I}{A^2}$$

BE CAREFUL ABOUT SIMPLIFICATION!!!



DOES NOT DEPEND ON SCALE!!!

(a) A set of rectangular sections with $\phi_B^e = 2$; (b) a set of I-sections with $\phi_B^e = 10$; and (c) a set of tubes with $\phi_B^e = 12$. Members of a set differ in size but not in shape.

ϕ_B^e the *shape factor for elastic bending*.

$$I_o = \frac{b_o^4}{12} = \frac{A^2}{12}$$

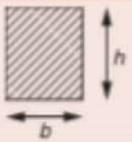
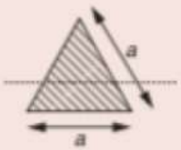
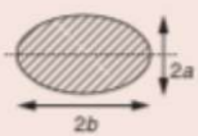
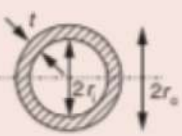
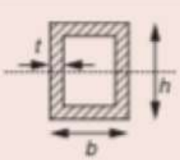
$$\boxed{\phi_B^e = \frac{S}{S_o} = \frac{EI}{EI_o} = \frac{12\,I}{A^2}}$$

$$\Phi_B^e = \frac{\overset{\text{rect.}}{I}}{\underset{\text{squ}}{I_o}} = \frac{\dfrac{b.h^3}{\cancel{12}}}{\dfrac{b_o^4}{\cancel{12}}}$$

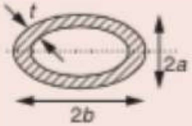
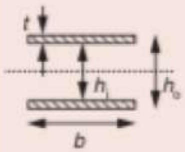
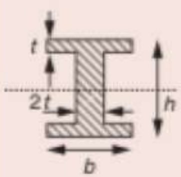
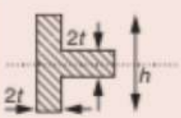
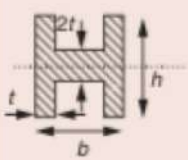
$$= \frac{bh^3}{b_o^4} = \frac{bh^3}{A_o^2} = \frac{(bh)^2 \cdot h/b}{A_o^2}$$

$$= \frac{h}{b}$$

Shape efficiency factors

Section shape	Bending factor, φ_B^e	Torsional factor, φ_T^e	Bending factor, φ_B^f	Torsional factor, φ_T^f	Bending factor, φ_B^{pl}
	$\frac{h}{b}$	$2.38\frac{h}{b}\left(1-0.58\frac{b}{h}\right)$ $(h > b)$	$\left(\frac{h}{b}\right)^{0.5}$	$1.6\sqrt{\frac{b}{h}}\frac{1}{h(1+0.6b/h)}$ $(h > b)$	$\left(\frac{h}{b}\right)^{0.5}$
	$\frac{2}{\sqrt{3}} = 1.15$	0.832	$\frac{3^{1/4}}{2} = 0.658$	0.83	$\frac{3^{1/4}}{2} = 0.658$
	$\frac{3}{\pi} = 0.955$	1.14	$\frac{3}{2\sqrt{\pi}} = 0.846$	1.35	$\frac{4}{3\sqrt{\pi}} = 0.752$
	$\frac{3}{\pi}\frac{a}{b}$	$\frac{2.28ab}{(a^2+b^2)}$	$\frac{3}{2\sqrt{\pi}}\sqrt{\frac{a}{b}}$	$1.35\sqrt{\frac{a}{b}}$ $(a < b)$	$\frac{4}{3\sqrt{\pi}}\sqrt{\frac{a}{b}} = 0.752\sqrt{\frac{a}{b}}$
	$\frac{3}{\pi}\left(\frac{r}{t}\right)$ $(r \gg t)$	$1.14\left(\frac{r}{t}\right)$	$\frac{3}{\sqrt{2\pi}}\sqrt{\frac{r}{t}}$	$1.91\sqrt{\frac{r}{t}}$	$\sqrt{\frac{2}{\pi}}\sqrt{\frac{r}{t}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ $(h, b \gg t)$	$\frac{3.57b^2(1-t/h)^4}{th(1+b/h)^3}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\frac{(1+3b/h)}{(1+b/h)^{3/2}}$	$3.39\sqrt{\frac{h^2}{bt}}\frac{1}{(1+h/b)^{3/2}}$	$\sqrt{2}\sqrt{\frac{h^2}{bt}}\frac{(1+h/2b)}{(1+h/b)^{3/2}}$

Solid equiaxed sections (circles, squares, hexagons, octagons) all have values very close to 1 – for practical purposes, they can be set equal to 1.

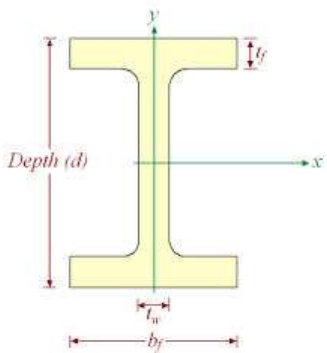
	$\frac{3}{\pi}\frac{a}{t}\frac{(1+3b/a)}{(1+b/a)^2}$ $(a,b \gg t)$	$\frac{9.12(ab)^{5/2}}{t(a^2+b^2)(a+b)^2}$	$\frac{3}{2\sqrt{\pi}}\sqrt{\frac{a}{t}}\frac{(1+3b/a)}{(1+b/a)^{3/2}}$	$5.41\sqrt{\frac{a}{t}}\frac{1}{(1+a/b)^{3/2}}$	$\frac{4}{\sqrt{\pi}}\sqrt{\frac{a^2}{bt}}\frac{(2+a/b)}{(1+a/b)^{3/2}}$
	$\frac{3}{2}\frac{h_o^2}{bt}(h, b \gg t)$	—	$\frac{3}{\sqrt{2}}\frac{h_o}{\sqrt{bt}}$	—	$\sqrt{2}\frac{h_o}{\sqrt{bt}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+3b/h)}{(1+b/h)^2}$ $(h, b \gg t)$	$1.19\left(\frac{t}{b}\right)\frac{(1+4h/b)}{(1+h/b)^2}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\frac{(1+3b/h)}{(1+b/h)^{3/2}}$	$1.13\sqrt{\frac{t}{b}}\frac{(1+4h/b)}{(1+h/b)^{3/2}}$	$\sqrt{2}\sqrt{\frac{h^2}{bt}}\frac{(1+h/2b)}{(1+h/b)^{3/2}}$
	$\frac{1}{2}\frac{h}{t}\frac{(1+4bt^2/h^3)}{(1+b/h)^2}$ $(h, b \gg t)$	$0.595\left(\frac{t}{h}\right)\frac{(1+8b/h)}{(1+b/h)^2}$	$\frac{3}{4}\sqrt{\frac{h}{t}}\frac{(1+4bt^2/h^3)}{(1+b/h)^{3/2}}$	$0.565\sqrt{\frac{t}{h}}\frac{(1+8b/h)}{(1+b/h)^{3/2}}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\left(\frac{h}{(h+b)}\right)^{3/2}\left[1+\frac{2t(b-2t)}{h^2}\right]$
	$\frac{1}{2}\frac{h}{t}\frac{(1+4bt^2/h^3)}{(1+b/h)^2}$ $(h, b \gg t)$	$1.19\left(\frac{t}{h}\right)\frac{(1+4b/h)}{(1+b/h)^2}$	$\frac{3}{4}\sqrt{\frac{h}{t}}\frac{(1+4bt^2/h^3)}{(1+b/h)^{3/2}}$	$1.13\sqrt{\frac{t}{h}}\frac{(1+4b/h)}{(1+b/h)^{3/2}}$	$\frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\left(\frac{h}{(h+b)}\right)^{3/2}\left[1+\frac{2t(b-2t)}{h^2}\right]$

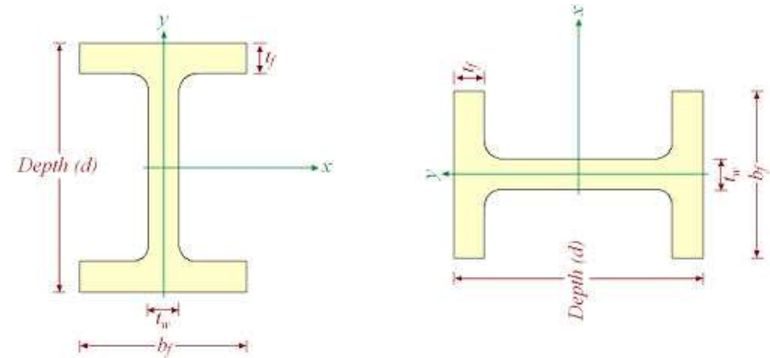
Structural Elements

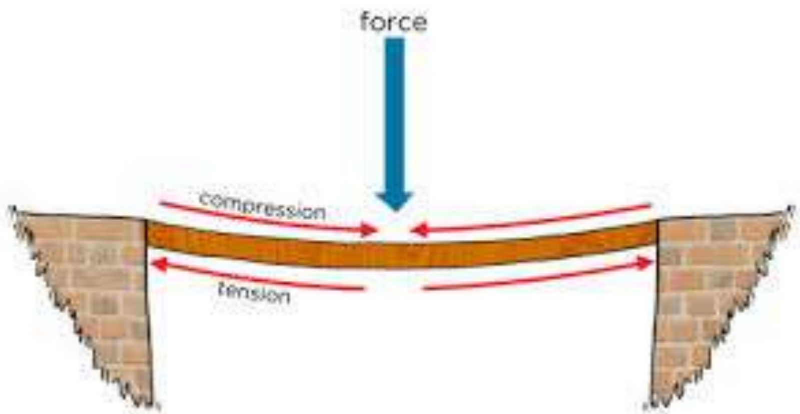
I - Beam

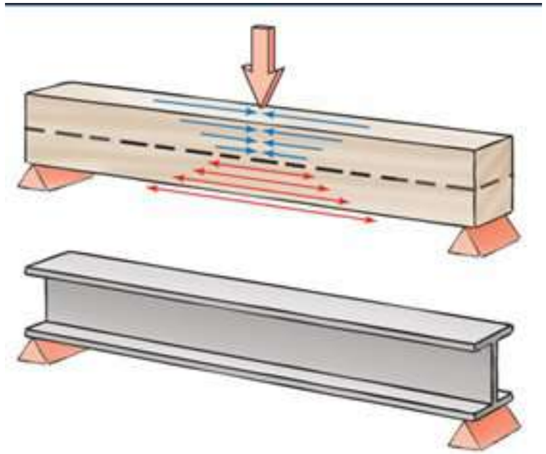
In which direction should it be loaded?

Why in I shape?









Is stiffness always required?

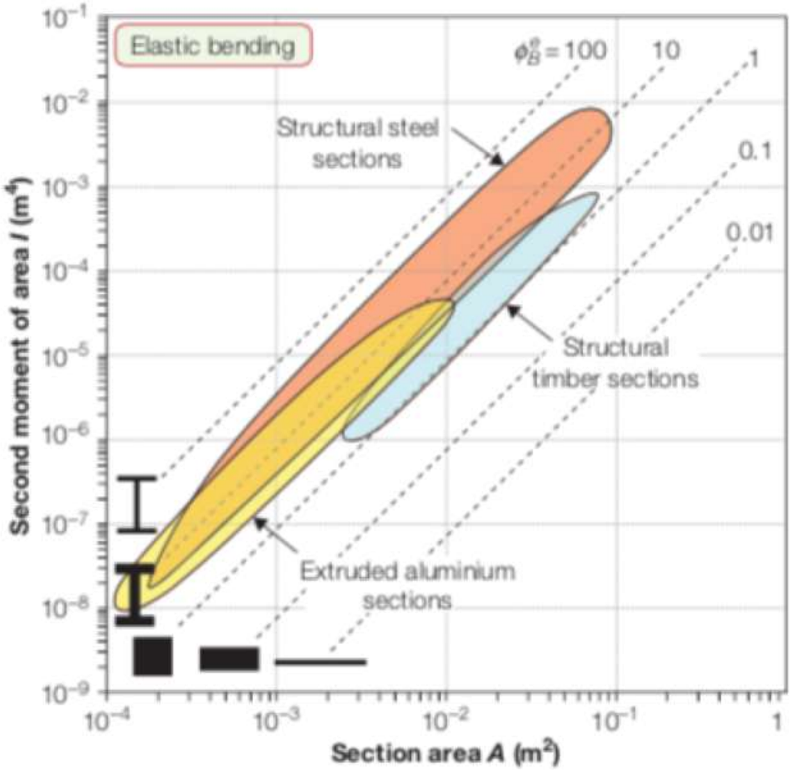
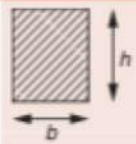
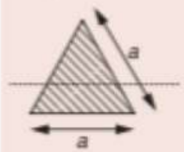
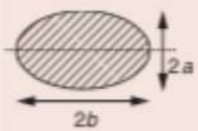
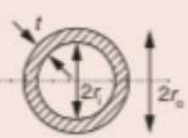
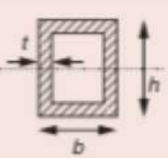


FIGURE 9.6

The second moment of area I plotted against section area A. Efficient structures have high values of the ratio I/A^2; inefficient structures (ones that bend easily) have low values. Real structural sections have values of I and A that lie in the shaded zones. Note that there are limits on A and on the maximum shape efficiency ϕ_B^e that depend on material.

Elastic Twisting of Shafts

Moments of sections, and units

Section shape	Area A (m)	Moment K (m^4)
	bh	$\frac{bh^3}{3}\left(1-0.58\frac{b}{h}\right)$ $(h > b)$
	$\frac{\sqrt{3}}{4}a^2$	$\frac{\sqrt{3}\,a^4}{80}$
	πr^2	$\frac{\pi}{2}r^4$
	πab	$\frac{\pi a^3 b^3}{(a^2+b^2)}$
	$\pi(r_o^2 - r_i^2)$ $\approx 2\pi rt$	$\frac{\pi}{2}(r_o^4 - r_i^4)$ $\approx 2\pi r^3 t$
	$2t(h+b)$ $(h, b \gg t)$	$\frac{2tb^2h^2}{(h+b)}\left(1-\frac{t}{h}\right)^4$

Shape Factor for Elastic Twisting

The shape factor for elastic twisting is defined, as before, by the ratio of the torsional stiffness of the shaped section, S_T, to that, S_{T_o}, of a solid square shaft of the same length L and cross-section A, which, using equation (11.5), is:

$$\phi_T^e = \frac{S_T}{S_{T_o}} = \frac{K}{K_o} \tag{11.6}$$

The torsional constant K_o for a solid square section (Table 11.1, top row with $b = h$) is

$$K_o = 0.14A^2$$

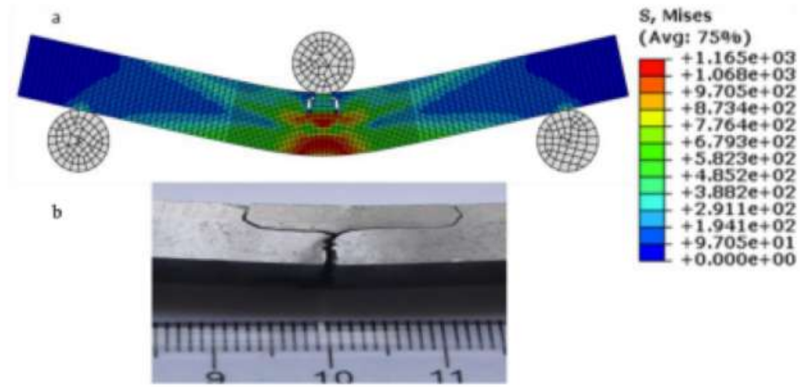
giving

$$\phi_T^e = 7.14\frac{K}{A^2} \tag{11.7}$$

What about failure?

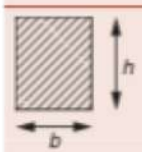
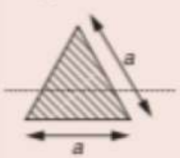
$\boxed{\phi^{e}_{B}}$ ϕ^{e}_{T} $\boxed{\phi^{f}_{B}}$

$$\sigma = \frac{M y_m}{I} = \frac{M}{Z}$$

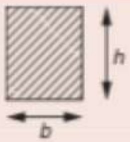
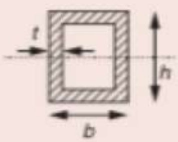


Three-Point Bending Fracture Properties of Multilayer Metal Hot Forging Die Specimen, Huajun Wang et al 2019 IOP Conf. Ser.: Mater. Sci. Eng. 472 012033

Moments of sections, and units

Section shape	Area A (m)	Moment I (m^4)	Moment K (m^4)	Moment Z (m^4)	Moment Q (m^4)	Moment Z_p (m^4)
	bh	$\frac{bh^3}{12}$	$\frac{bh^3}{3}\left(1 - 0.58\frac{b}{h}\right)$ $(h > b)$	$\frac{bh^2}{6}$	$\frac{b^2h^2}{(3h + 1.8b)}$ $(h > b)$	$\frac{bh^2}{4}$
	$\frac{\sqrt{3}}{4}a^2$	$\frac{a^4}{32\sqrt{3}}$	$\frac{\sqrt{3}\,a^4}{80}$	$\frac{a^3}{32}$	$\frac{a^3}{20}$	$\frac{3a^3}{64}$

Moments of sections, and units

Section shape	Area A (m)	Moment I (m^4)	Moment K (m^4)	Moment Z (m^4)	Moment Q (m^4)	Moment Z_p (m^4)
	bh	$\frac{bh^3}{12}$	$\frac{bh^3}{3}\left(1-0.58\frac{b}{h}\right)$ $(h>b)$	$\frac{bh^2}{6}$	$\frac{b^2h^2}{(3h+1.8b)}$ $(h>b)$	$\frac{bh^2}{4}$
	$2t(h+b)$ $(h, b \gg t)$	$\frac{1}{6}h^3t\left(1+3\frac{b}{h}\right)$	$\frac{2tb^2h^2}{(h+b)}\left(1-\frac{t}{h}\right)^4$	$\frac{1}{3}h^2t\left(1+3\frac{b}{h}\right)$	$2tbh\left(1-\frac{t}{h}\right)^2$	$bht\left(1+\frac{h}{2b}\right)$

Strength Efficiency

The strength-efficiency of the shaped beam, ϕ_B^f, is measured by the ratio Z/Z_o, where Z_o is the section modulus of a reference beam of square section with the same cross-sectional area, A:

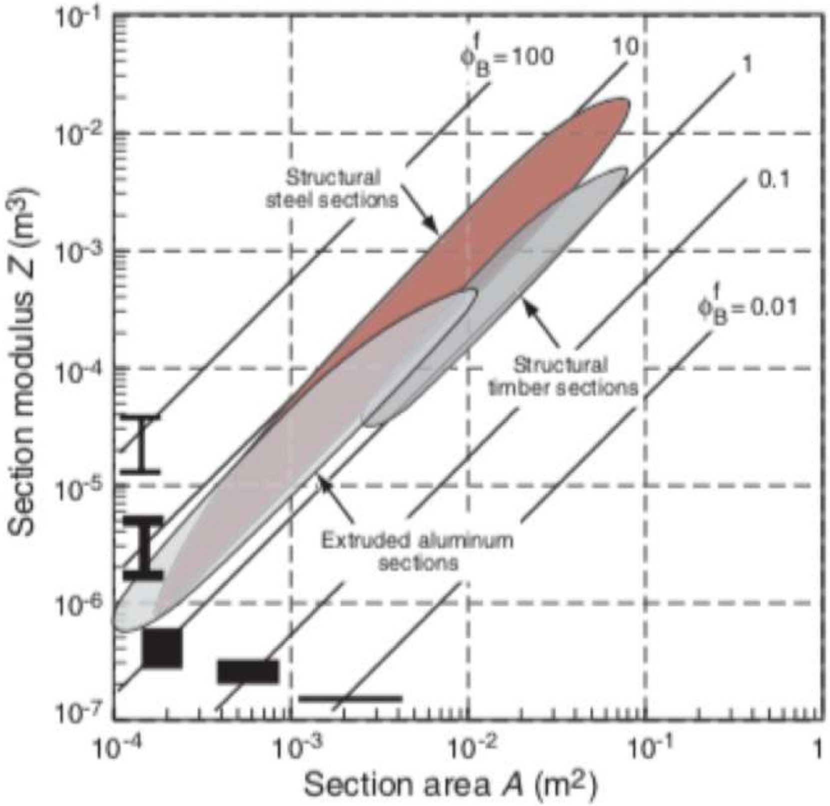
$$Z_o = \frac{b_o^3}{6} = \frac{A^{3/2}}{6} \qquad (11.9)$$

Thus:

$$\boxed{\phi_B^f = \frac{Z}{Z_o} = \frac{6Z}{A^{3/2}}} \qquad (11.10)$$

Like the other shape-efficiency factor, it is dimensionless and therefore independent of scale, and its value for a beam with a solid square section is unity. Table 11.3 gives expressions for other shapes derived from the values of the section modulus Z, which can be found in Table 11.2. A beam with an failure shape-efficiency factor of 10 is 10 times stronger in bending than a solid square section of the same weight.

Section Modulus



A beam with a failure shape efficiency factor of 10 is 10 times stronger in bending than a solid square section of the same weight.

The section modulus, Z, plotted against section area A. Efficient structures have high values of the ratio $Z/A^{3/2}$; inefficient structures (ones that bend easily) have low values. Real structural sections have values of Z and A that lie in the shaded zones. Note that there are limits on A and on the maximum shape efficiency ϕ_B^f that depend on material.

Evaluating shape factors

A beam has a square-box section with a height $h = 100$ mm, a width $b = 100$ mm, and a wall thickness $t = 5$ mm. What is the value of its shape factor ϕ_B^f?

Evaluating shape factors

A beam has a square-box section with a height $h = 100$ mm, a width $b = 100$ mm, and a wall thickness $t = 5$ mm. What is the value of its shape factor ϕ_B^f?

Answer

The shape factor for the box section, from Table 9.3, is $\phi_B^f = \frac{1}{\sqrt{2}}\sqrt{\frac{h}{t}}\frac{\left(1+\frac{3b}{h}\right)}{\left(1+\frac{b}{h}\right)^{3/2}} = 4.47$. The box section is stronger than a solid square-section beam of the same mass per unit length by a factor of 4.5.

Failure in Torsion

$$\tau = \frac{T r_m}{J}$$

$$\tau = \frac{T}{Q}$$

$$\phi_T^f = \frac{Q}{Q_o} = 4.8 \frac{Q}{A^{3/2}}$$

τ = Torsional stress

T = Torque

r_m = radial distance

J = Torsional moment of area

$Q = J/r_m$

Limits to Shape Efficiency

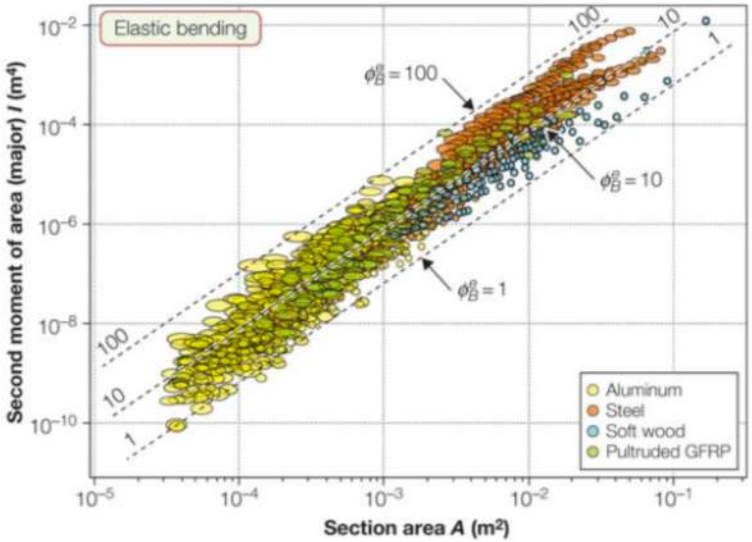


FIGURE 9.8
Log (I) plotted against log (A) for standard sections of steel, aluminum, pultruded GFRP, and wood. Contours of ϕ_B^e are shown, illustrating that there is an upper limit. A similar plot for log (Z) against log (A) reveals an upper limit for ϕ_B^f.

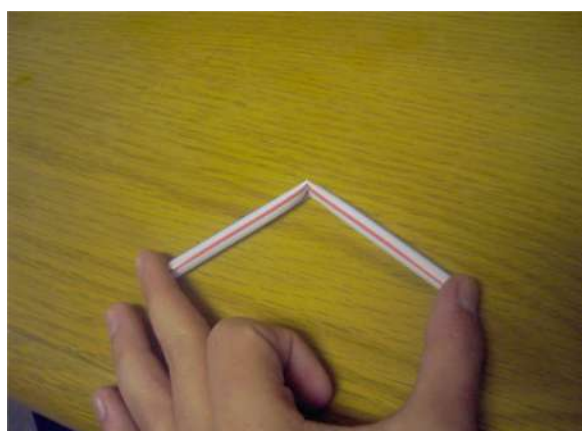
Empirical upper limits for the shape factors $\phi_B^e, \phi_T^e, \phi_B^f$ and ϕ_T^f

Material	$(\phi_B^e)_{max}$	$(\phi_T^e)_{max}$	$(\phi_B^f)_{max}$	$(\phi_T^f)_{max}$
Structural steel	65	25	13	7
6061 aluminum alloy	44	31	10	8
GFRP and CFRP	39	26	9	7
Polymers (e.g. nylons)	12	8	5	4
Woods (solid sections)	5	1	3	1
Elastomers	< 6	3	–	–

Limits to Shape Efficiency

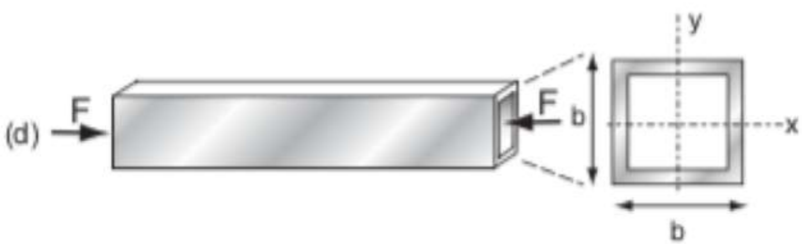
Empirical limits.

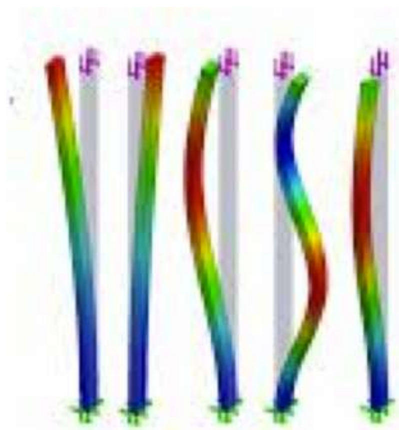
Limits imposed by local buckling.



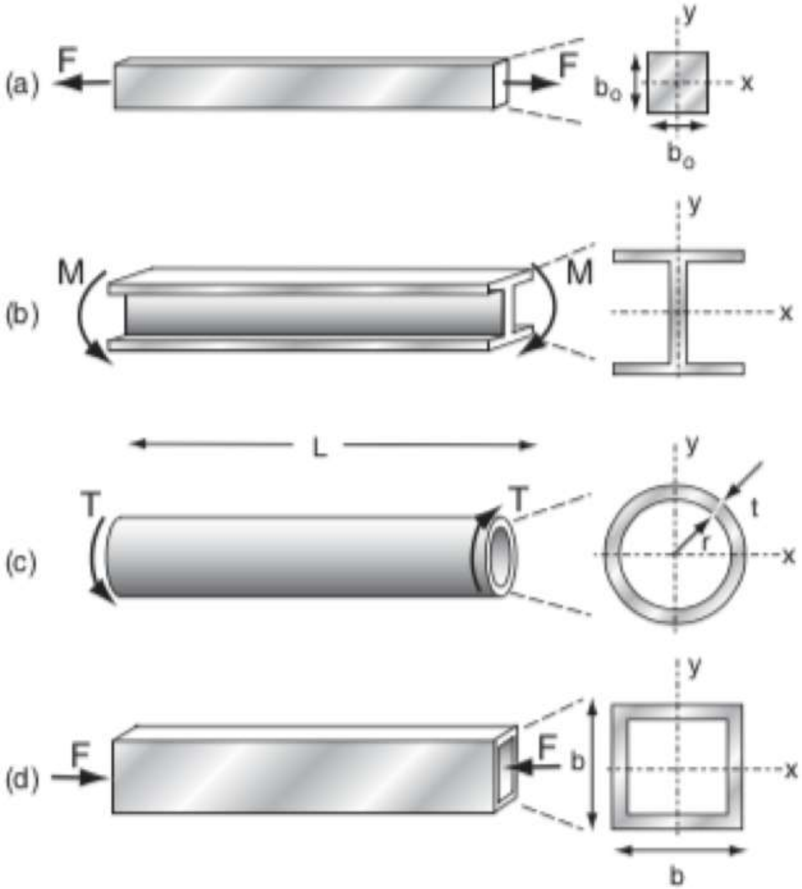
Buckling

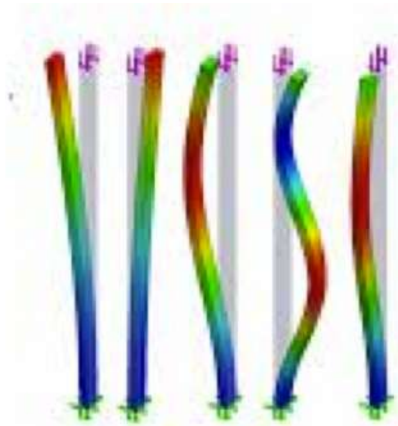
A column of length L, loaded in compression, buckles elastically when the load exceeds the Euler load.





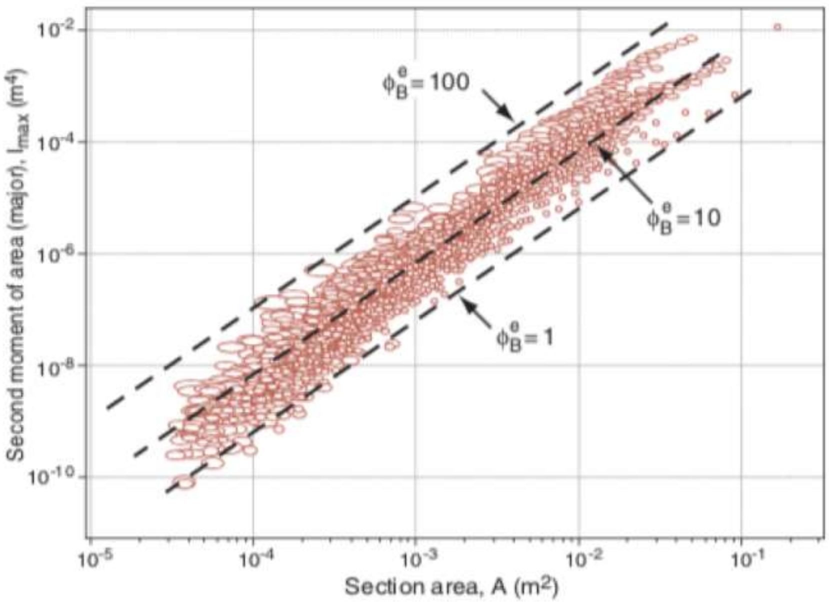
Buckling





$$F_c = \frac{n^2 \pi^2 E I_{\min}}{L^2}$$

Limits to Shape Efficiency



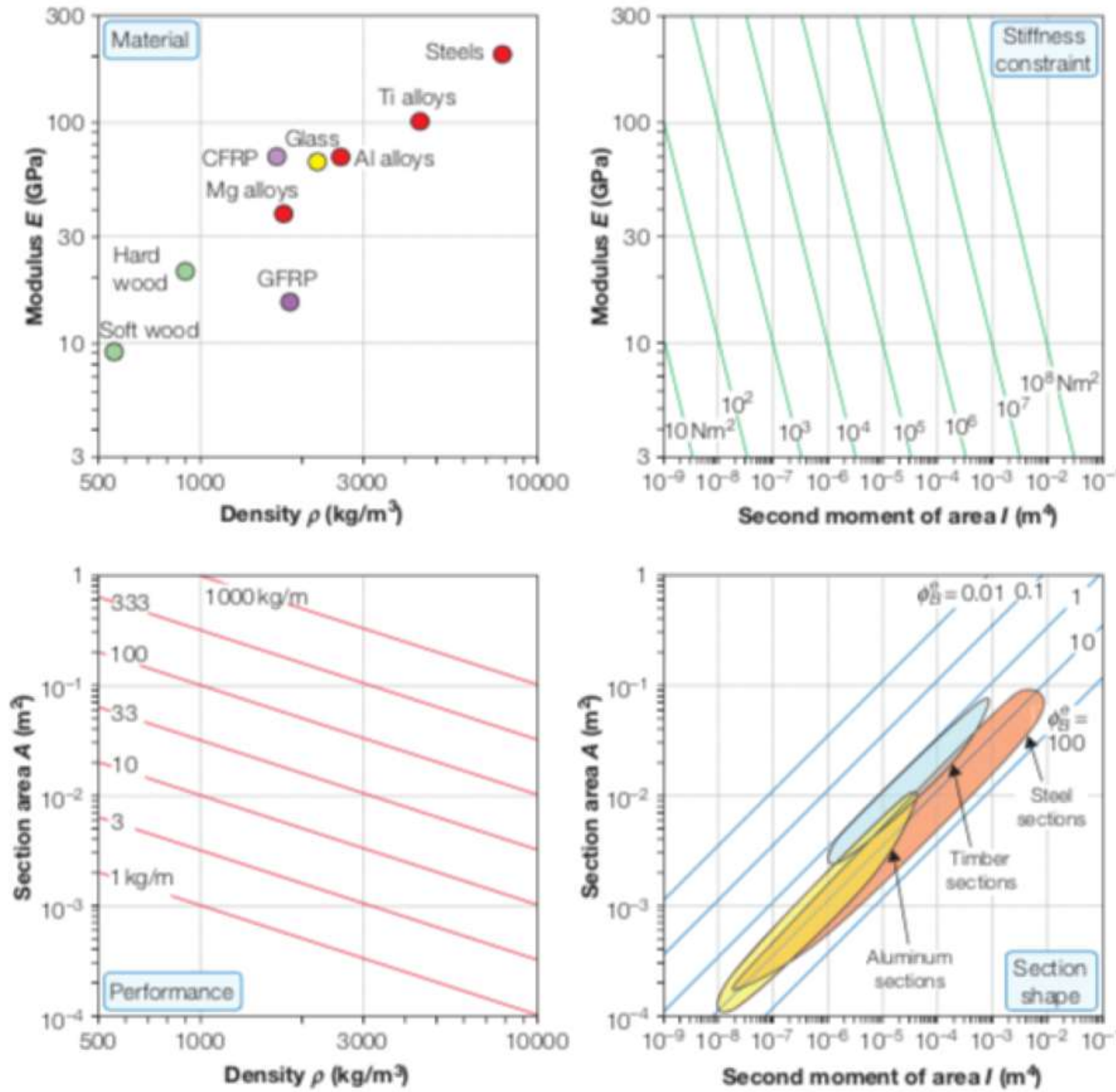
Limits

$$(\phi_B^e)_{max} \approx 2.3 \left(\frac{E}{\sigma_f}\right)^{1/2}$$

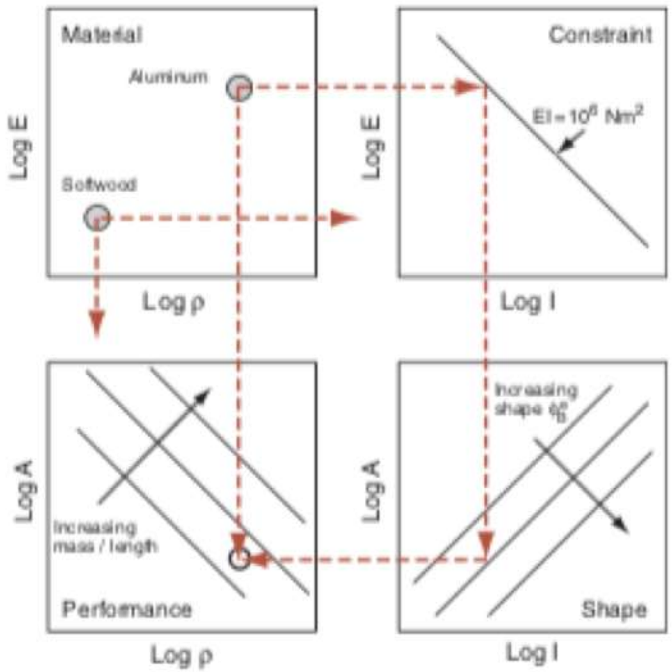
$$(\phi_B^f)_{max} \approx \sqrt{(\phi_B^e)_{max}}$$

Exploring Material Shape Combination

Stiffness-limited design.

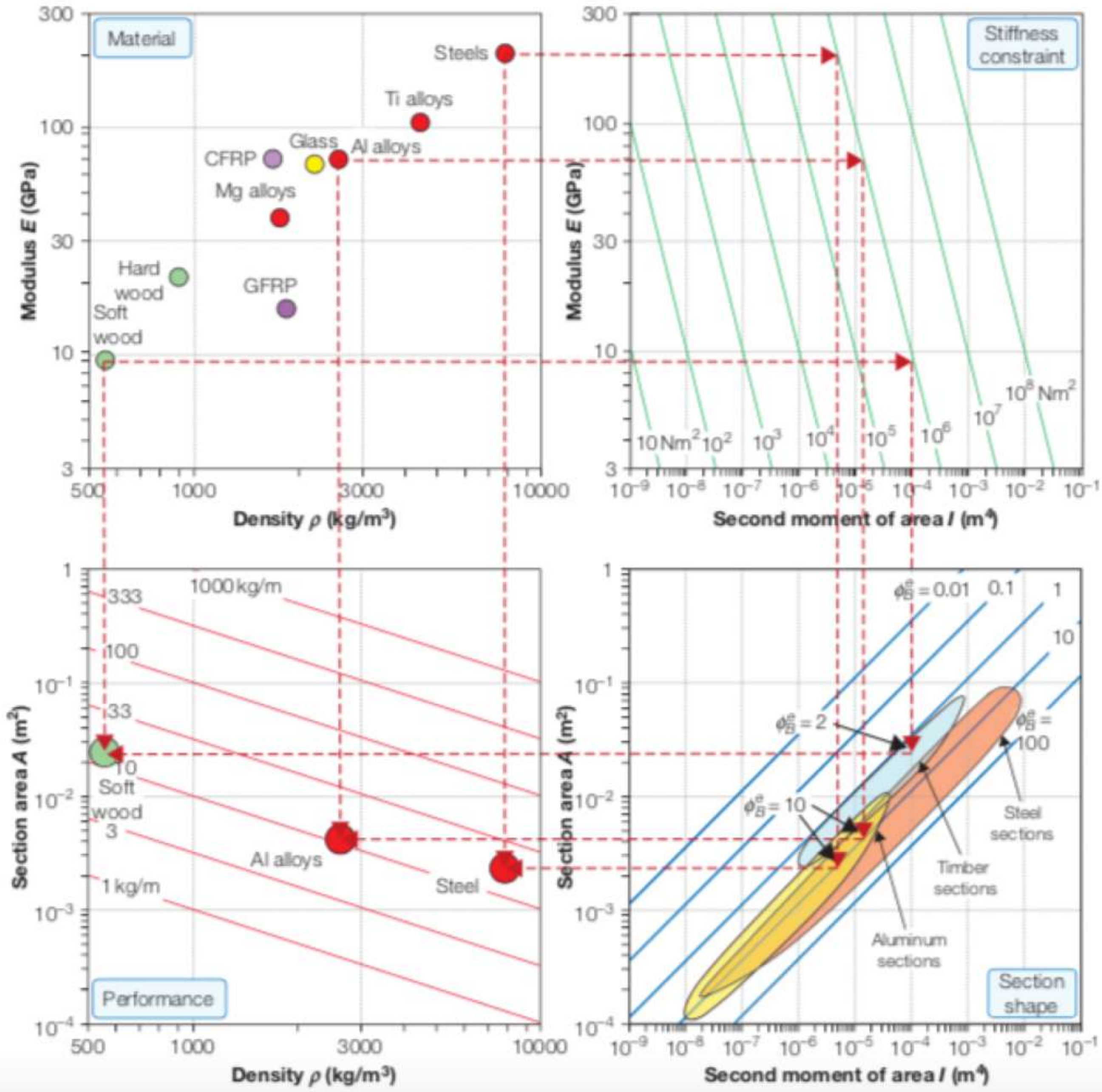


Exploring Material Shape Combination



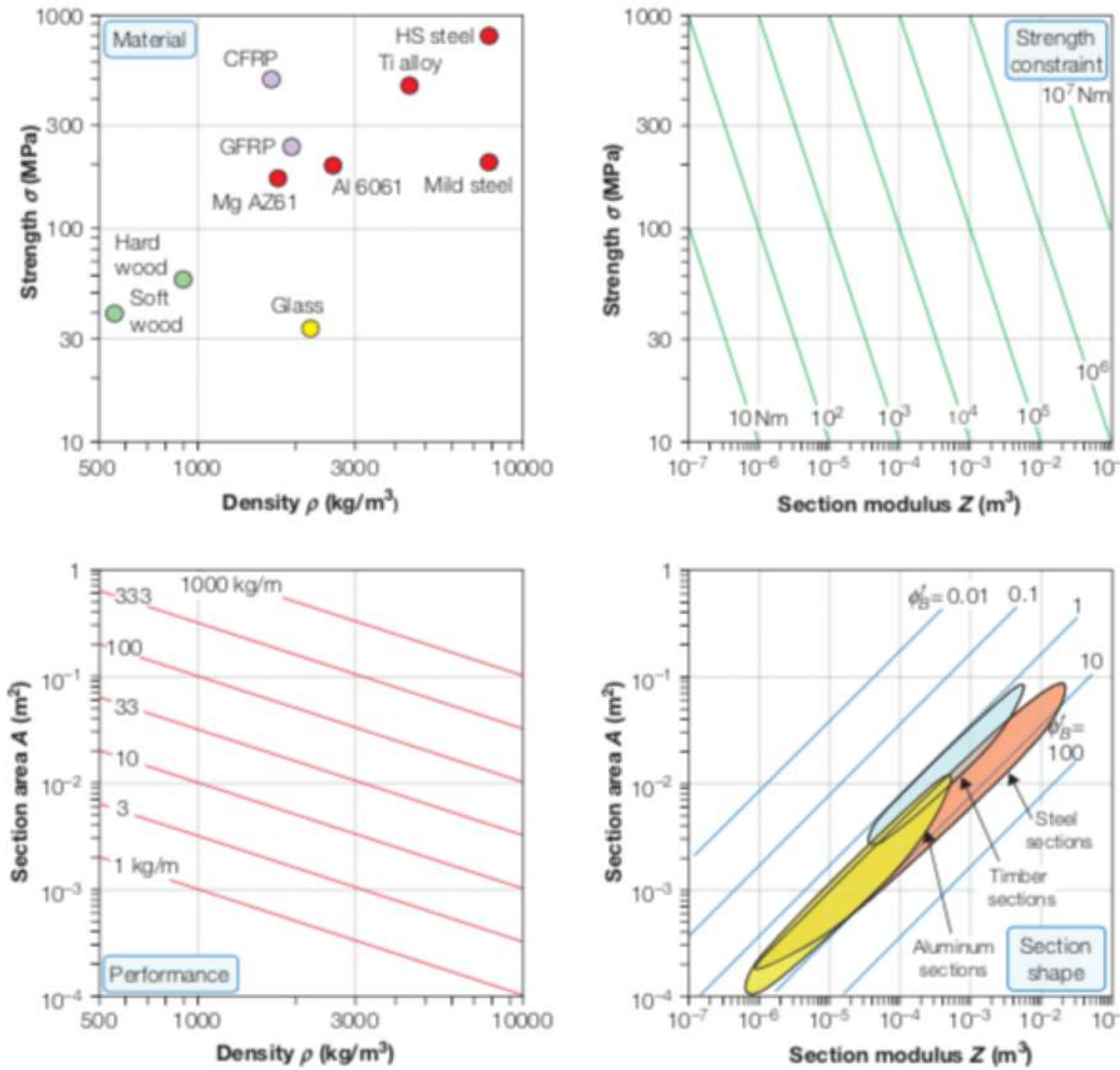
- Chose a material for the section and mark its modulus E and density ρ onto the *materials chart* in the first quadrant of the figure.
- Choose the desired section stiffness (EI); it is a constraint that must be met by the section. Extend a horizontal line from the value of E for the material to the appropriate contour in the *constraint chart* in the second quadrant.
- Drop a vertical from this point onto the *shape chart* in the third quadrant to the line describing the shape factor ϕ_B^e for the section. Values of I and A outside the shaded bands are forbidden.
- Extend a horizontal line from this point to the *performance chart* in the final quadrant (the one on the bottom left). Drop a vertical from the material density ρ in the material chart. The intersection shows the mass per unit length of the section.

Exploring Material Shape Combination



Exploring Material Shape Combination

Strength-limited design.



Material Indices That Include Shape

$$m = AL\rho$$

Its bending stiffness is

$$S_B = C_1 \frac{EI}{L^3} \tag{11.25}$$

where C_1 is a constant that depends only on the way the loads are distributed on the beam. Replacing I by ϕ_B^e using equation (11.3) gives

$$S_B = \frac{C_1}{12} \frac{E}{L^3} \phi_B^e A^2 \tag{11.26}$$

Using this to eliminate A in equation (11.24) gives the mass of the beam.

$$m = \left(\frac{12 S_B}{C_1}\right)^{1/2} L^{5/2} \left[\frac{\rho}{(\phi_B^e E)^{1/2}}\right] \tag{11.27}$$

$$M_1 = \frac{(\phi_B^e E)^{1/2}}{\rho}$$

Material Indices That Include Shape

The selection of material and shape for a light, stiff, beam

Material	ρ (Mg/m^3)	E (GPa)	ϕ_B^e	$E^{1/2}/\rho$	$(\phi_B^e E)^{1/2}/\rho$
1020 Steel	7.85	205	20	1.8	8.2
6061-T4 Al	2.7	70	15	3.1	**12.0**
GFRP (isotropic)	1.75	28	8	2.9	8.5
Wood (oak)	0.9	13.5	2	**4.1**	5.8

Material Indices That Include Shape

$$S_T = \frac{KG}{L}$$

where G is the shear modulus. Replacing K by ϕ_T^e using equation (11.7) gives

$$S_T = \frac{G}{7.14L}\phi_T^e A^2 \tag{11.30}$$

Using this to eliminate A in equation (11.24) gives

$$m = \left(7.14\frac{S_T}{L^3}\right)^{1/2} L^{3/2}\left[\frac{\rho}{(\phi_T^e G)^{1/2}}\right] \tag{11.31}$$

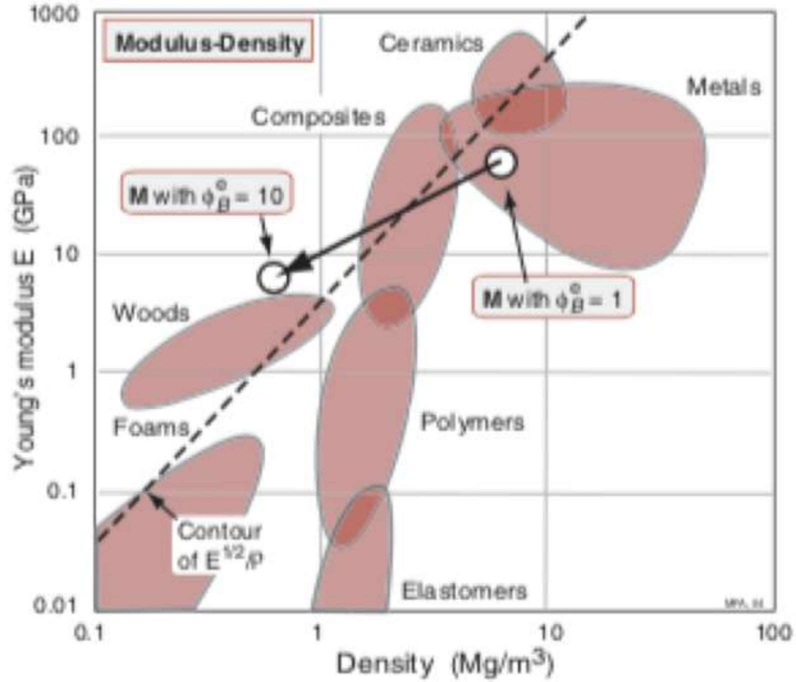
The best material-and-shape combination is that with the greatest value of $(\phi_T^e G)/\rho^{1/2}$. The shear modulus, G, is closely related to Young's modulus E. For the practical purposes we approximate G by $3/8E$; when the index becomes

$$M_2 = \frac{(\phi_T^e E)^{1/2}}{\rho} \tag{11.32}$$

Graphical Coselecting Using Indices

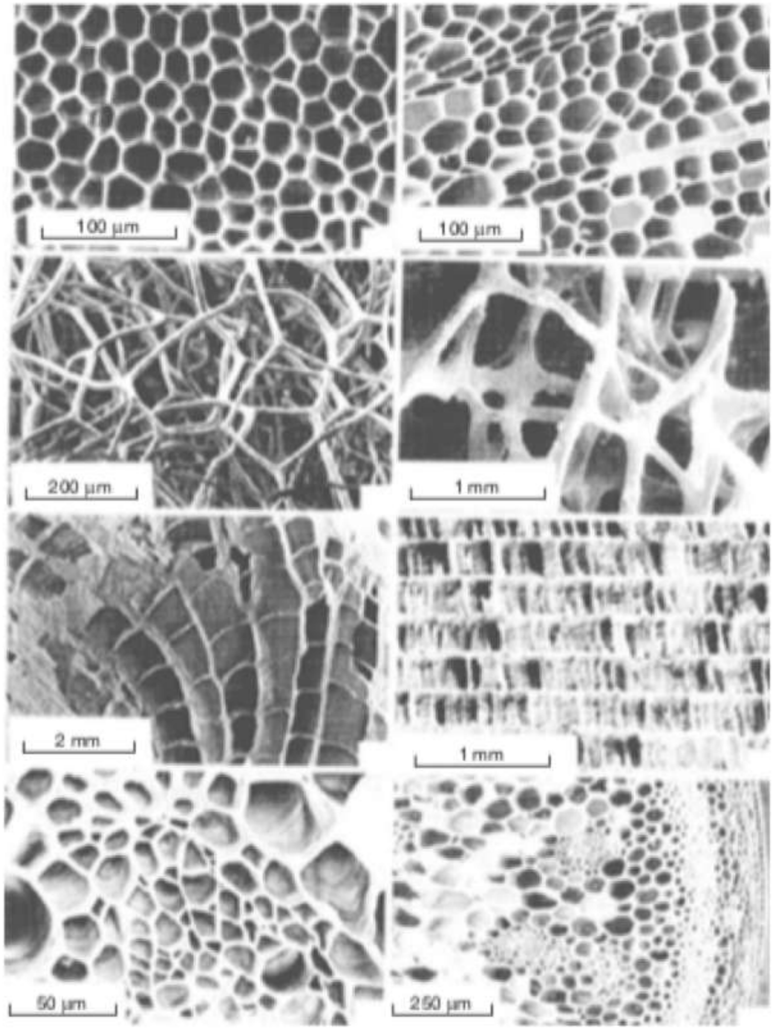
$$M_1 = \frac{(\phi_B^e E)^{1/2}}{\rho} = \frac{(E/\phi_B^e)^{1/2}}{\rho/\phi_B^e} = \frac{E^{*\,1/2}}{\rho^*}$$

$$E^* = \frac{E}{\phi_B^e} \quad \text{and} \quad \rho^* = \frac{\rho}{\phi_B^e}$$

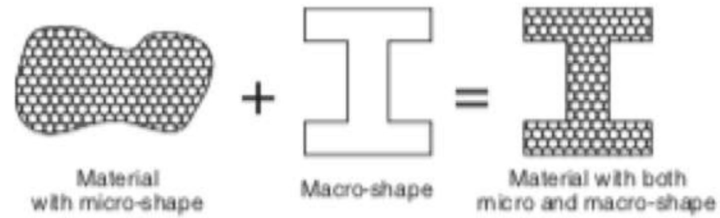


16 The structured material behaves like a new material with a modulus $E^* = E/\phi_B^e$ and a density $\rho^* = \rho/\phi_B^e$, moving it from a position below the broken selection line to one above. A similar procedure can be applied for bending strength, as described in the text.

Architectured Materials: Microscopic Shape

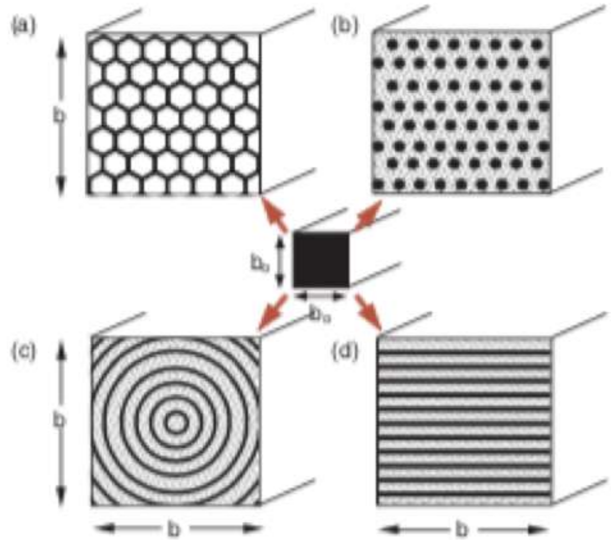


Natural materials with internal, or microscopic, shape. Reading from the top left: cork, balsa wood, sponge, cancellous bone, coral, cuttle-bone, and palm plant stalk.



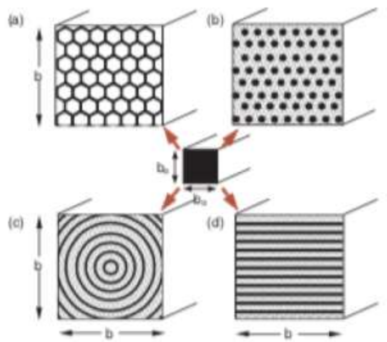
Micro-structural shape can be combined with macroscopic shape to give efficient structures. The overall shape factor is the product of the microscopic and macroscopic shape factors.

Architectured Materials: Microscopic Shape



.10 Four extensive micro-structured materials that are mechanically efficient: (a) prismatic cells, (b) fibers embedded in a foamed matrix, (c) concentric cylindrical shells with foam between, and (d) parallel plates separated by foamed spacers.

Architectured Materials: Microscopic Shape



.10 Four extensive micro-structured materials that are mechanically efficient: (a) prismatic cells, (b) fibers embedded in a foamed matrix, (c) concentric cylindrical shells with foam between, and (d) parallel plates separated by foamed spacers.

$$S_s \propto E_s I_s$$

$$b = \left(\frac{\rho_s}{\rho}\right)^{1/2} b_o$$

$$I = \frac{b^4}{12} = \frac{1}{12}\left(\frac{\rho_s}{\rho}\right)^2 b_o^4 = \left(\frac{\rho_s}{\rho}\right)^2 I_s$$

$$E = \left(\frac{\rho}{\rho_s}\right) E_s \qquad \psi_B^e = \frac{S}{S_s} = \frac{EI}{E_s I_s} = \frac{\rho_s}{\rho}$$

ψ_B^e as the *microscopic shape factor for elastic bending*

Architectured Materials: Microscopic Shape