ESKİŞEHİR TEKNİK ÜNİVERSİTESİ
ESKİŞEHİR TECHNICAL UNIVERSITY

MATERIALS SCIENCE AND ENGINEERING

MLZ 460 Materials Selection & Design

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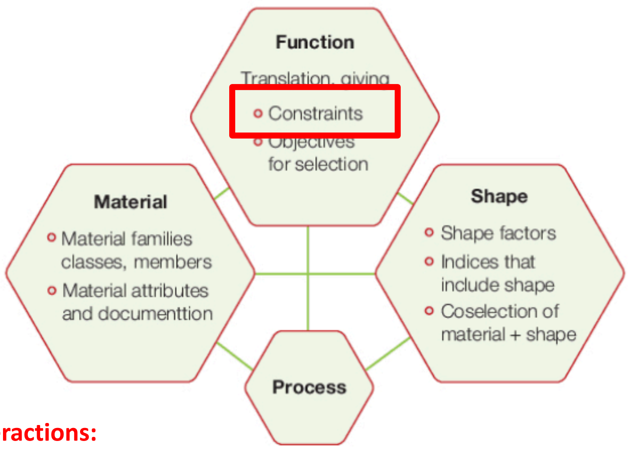
Multiple Constraints

The interaction between function, material, shape and process, lies at the heart of the material selection process!!!



Two-way interactions:

1st specification of shape restricts the choice of material and process but equally

2nd specification of process limits the material choice and the accessible shapes.

The more sophisticated the design, the tighter the specifications and the greater the interactions.

2

Most decisions in life involve trade-offs !!!

Is it possible to be rich and happy at the same time?

&

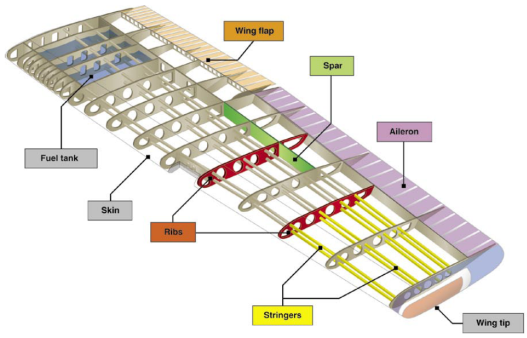
How can you measure it?

3

The selection must satisfy several, often conflicting, constraints

In case of one main Objective;
Ex. Weight;

Constraints:
- Stiffness,
- Fatigue strength
- Toughness
- Geometry



Aircraft Wing Spar

THE LIGHTEST MATERIAL WILL NOT BE THE CHEAPEST ONE !!!

4

The selection must satisfy several, often conflicting, constraints

In case of one main Objective;
Ex. Cost;

Constraints:
- Stiffness,
- Strength
- Thermal Conductivity

Disposable Hot-Drink Cup

THE CHEAPEST MATERIAL WILL NOT BE THE SAFE ONE !!!

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SELECTION WITH MULTIPLE CONSTRAINTS

IF THERE IS ONE MAIN OBJECTIVE (Minimization of weight or cost)

WITH MANY CONSTRAINTS

SOLUTION:

1. APPLY THE CONSTRAINTS IN SEQUENCE

2. REJECT THE MATERIALS THAT FAIL TO MEET THEM

3. FIND THE CANDIDATES (The survivors are viable candidates!)

4. RANK THEM BY THEIR ABILITY TO MEET THE SINGLE OBJECTIVE AND THEN EXPLORE THE TOP-RANKED CANDIDATES IN DETAIL ! & Make the final choice !!!

IN CASE OF TWO OR MORE OBJECTIVE ?

EX: WEIGHT & COST

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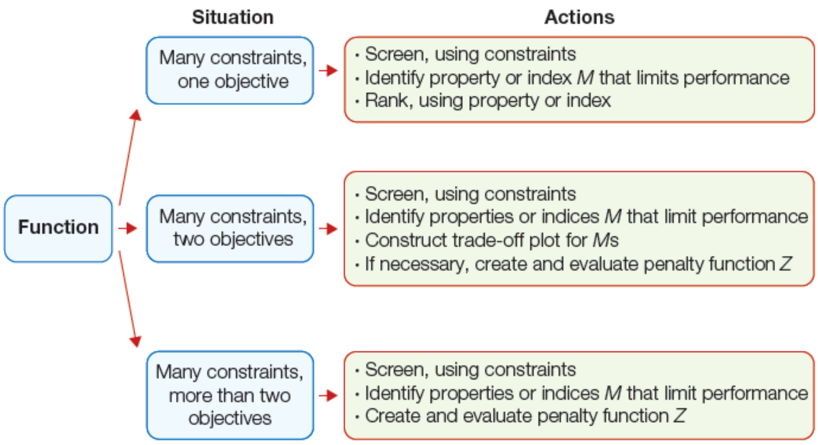
The choice of materials that best meets one objective will not usually be that which best meets others !

Strategies for tackling selection with multiple constraints and conflicting objectives.



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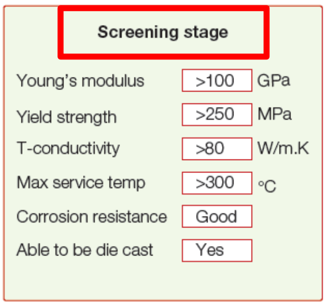
SELECTION WITH MULTIPLE CONSTRAINTS



(a)

This box represents screening by imposing constraints on properties, on requirements such as corrosion resistance, or on the ability to be processed in a certain way.

(b)

Here a bar chart for cost for the surviving candidate materials—indicates how they are ranked.

All very simple

THEN WHAT HAPPENS IF YOUR SINGLE OBJECTIVE IS LIMITED MORE THAN ONE CONSTRAINT

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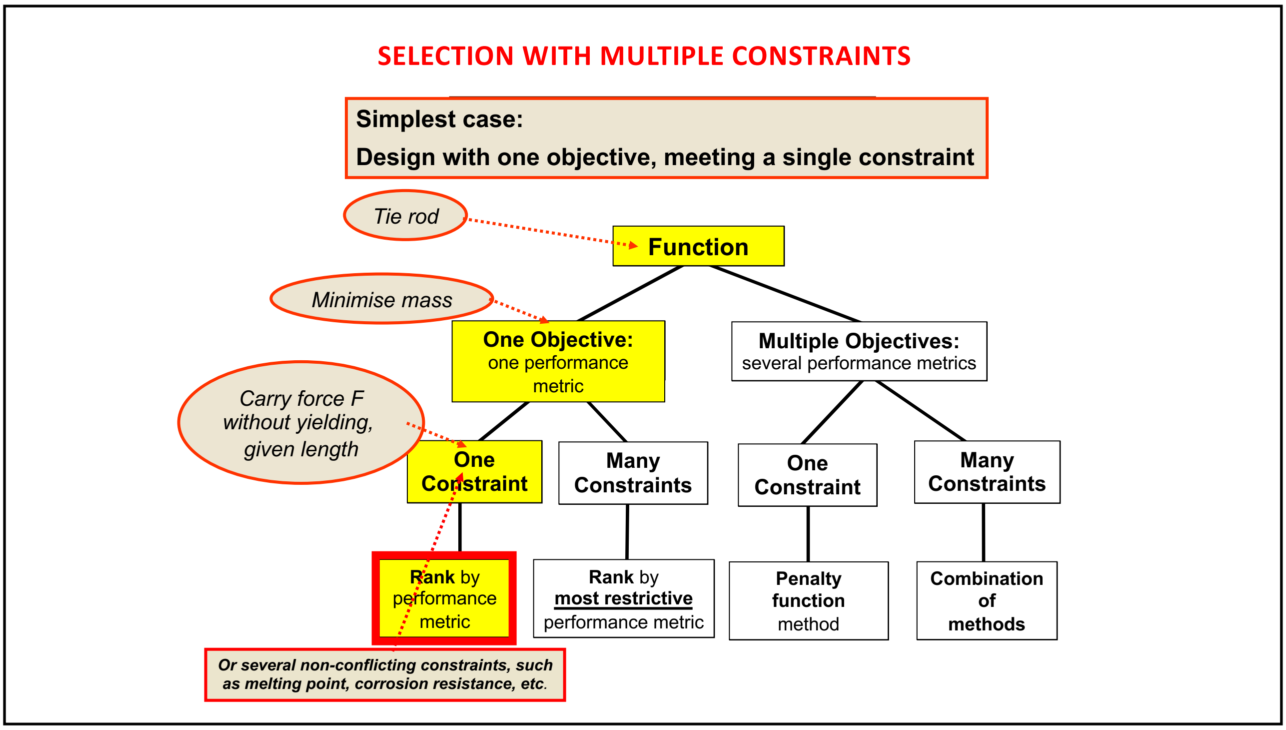
SELECTION WITH MULTIPLE CONSTRAINTS
Simplest case:
Design with one objective, meeting a single constraint
Tie rod
Function
Minimise mass
One Objective: one performance metric
Multiple Objectives: several performance metrics
Carry force F without yielding, given length
One Constraint
Many Constraints
One Constraint
Many Constraints
Rank by performance metric
Rank by most restrictive performance metric
Penalty function method
Combination of methods
Or several non-conflicting constraints, such as melting point, corrosion resistance, etc.

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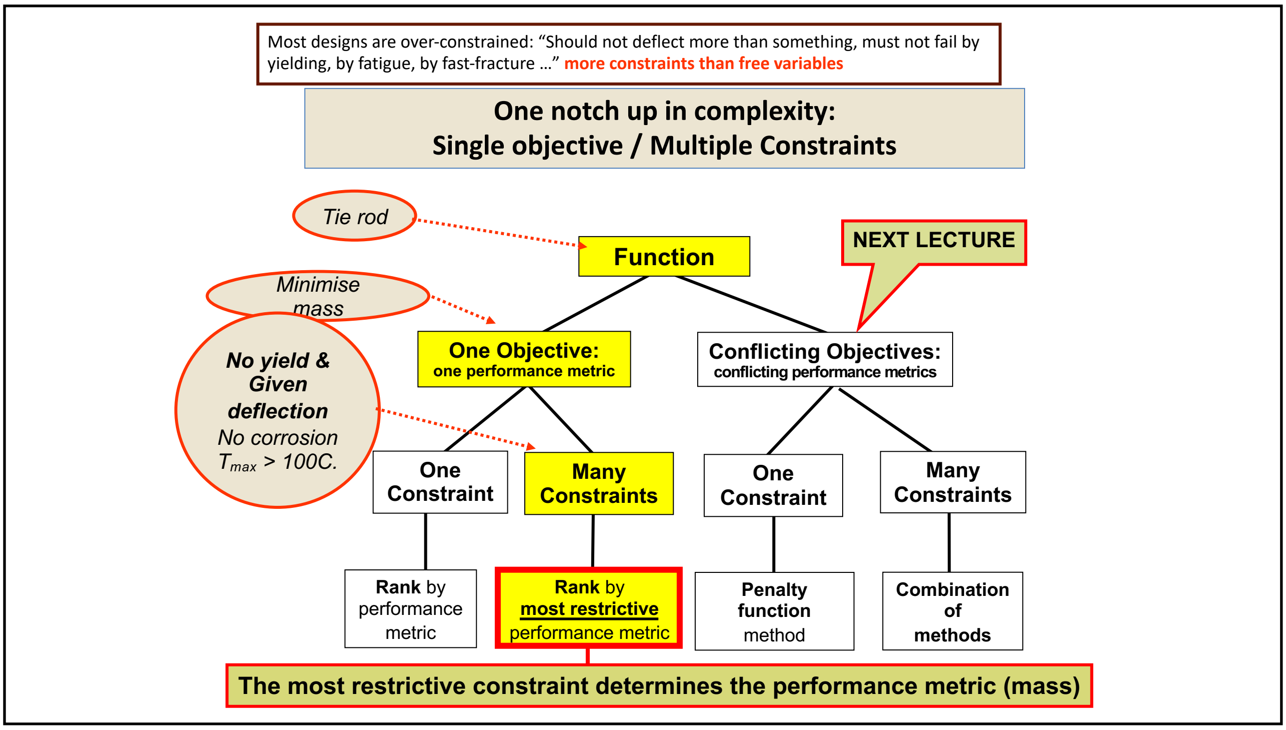
Most designs are over-constrained: "Should not deflect more than something, must not fail by yielding, by fatigue, by fast-fracture …" more constraints than free variables
One notch up in complexity:
Single objective / Multiple Constraints
Tie rod
Function
NEXT LECTURE
Minimise mass
One Objective: one performance metric
Conflicting Objectives: conflicting performance metrics
No yield & Given deflection
No corrosion
Tmax > 100C.
One Constraint
Many Constraints
One Constraint
Many Constraints
Rank by performance metric
Rank by most restrictive performance metric
Penalty function method
Combination of methods
The most restrictive constraint determines the performance metric (mass)

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EXAMPLE: Minimizing Mass (m): A light, stiff and strong tie-rod

A design calls for a tie-rod. It must carry a tensile force F* without failure and be as light as possible (Figure). The length L is specified but the cross-section area A is not. Here, “maximizing performance” means “minimizing the mass while still carrying the load F* safely.”

Strong tie of length L and minimum mass

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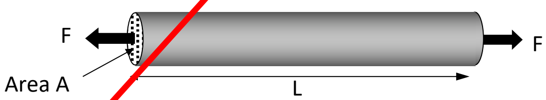
QUESTION: Materials for a stiff, light tie-rod — Constraint # 1

Strong tie of length L and minimum mass



Function — *Tie-rod*

Constraints
- *Length L is specified*
- *Must be stiff*

Equation for constraint on A:
$S = EA / L$ (1)

Objective (Goal)

Minimise mass m:
$m = A L \rho$ (2)

Free variables
- *Material choice*
- *Section area A*

Eliminate A in (2) using (1):

Performance metric m_1

$$m_1 = L^2 S\left(\frac{\rho}{E}\right)$$

Chose materials with smallest M1 = $\left(\frac{\rho}{E}\right)$

m = mass
A = cross-sec. area
L = length
ρ = density
E= elastic modulus
S = stiffness

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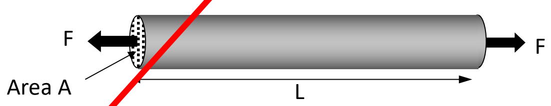
QUESTION: Materials for a strong, light tie-rod — **Constraint # 2**

Strong tie of length L and minimum mass



Function: *Tie-rod*

Constraints:
- *Length L is specified*
- *Must not fail under load F*

Equation for constraint on A:
$F/A < \sigma_y$ (1)

Objective (Goal):
Minimise mass m:
$m = A\,L\,\rho$ (2)

m = mass
A = cross-sec. area
L = length
ρ = density
σ_y = yield strength

Free variables:
- *Material choice*
- *Section area A*

Eliminate A in (2) using (1):

Performance metric m_2: $m_2 = LF\left(\frac{\rho}{\sigma_y}\right)$

Chose materials with smallest M2 = $\left(\frac{\rho}{\sigma_y}\right)$

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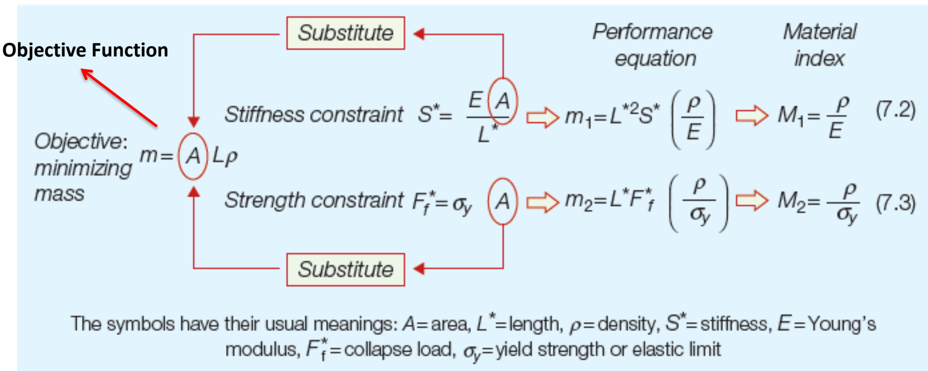
Tie-rod of minimum mass might specify both stiffness and strength, leading to two independent performance equations:

Objective Function



$$\tilde{m} = \max(m_1, m_2)$$

Search for the material that offers the smallest value of…

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Analytical Solution: (in 3 steps)

Rank by the more restrictive of the constraints

EXAMPLE: A material is required for a light tie of specified length **L**, stiffness **S**, and collapse load $\mathbf{F_f}$ with the values of;

$L^* = 1m \quad S^* = 3 \times 10^7$ N/m $\quad F_f^* = 10^5$ N

Material	ρ (kg/m^3)	E (GPa)	σ_y (MPa)
1020 Steel	7850	205	320
6061 Al	2700	70	120
Ti-6-4	4400	115	950

1. Calculate m_1 and m_2 for given L and F

$$m_1 = L^2 S\left(\frac{\rho}{E}\right) \qquad m_2 = LF\left(\frac{\rho}{\sigma_y}\right)$$

Material	ρ (kg/m^3)	E (GPa)	σ_y (MPa)	m_1 (kg)	m_2 (kg)	$\tilde{m}$ (kg)
1020 Steel	7850	205	320	1.15	2.45	2.45
6061 Al	2700	70	120	1.16	2.25	2.25
Ti-6-4	4400	115	950	1.15	0.46	1.15

2. Find the **largest** of every pair of m's
3. Find the **smallest** of the larger ones

The most restrictive constraint requires a larger mass and thus becomes the controlling or active constraint.

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3000 MATERIALS ???

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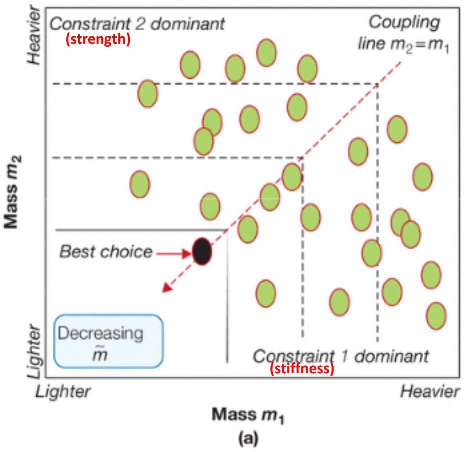
The Graphical Method



Each bubble represents a material

We wish to minimize mass;

Line m1=m2 separates the graph into 2 regions

m1 > m2 constraint 1 **(stiffness)** is dominant

m2 > m1 constraint 2 **(strength)** is dominant

This defines a box-shaped selection envelope with its corner on the m1=m2 line. The nearer the box is pulled to the bottom left, the smaller is ~m. The best choice is the last material left in the box.

This solution is obvious for specific S^*, L^* and F_f^* values.

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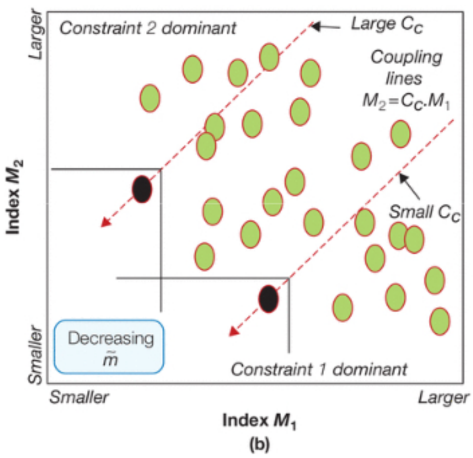
The Graphical Method

Suppose, instead, that we plot the material indices M1 and M2 that are contained in the performance equations, as shown above figure.

Performance equation	Material index
$m_1 = L^{*2}S^*\left(\frac{\rho}{E}\right)$	$M_1 = \frac{\rho}{E}$
$m_2 = L^*F^*_f\left(\frac{\rho}{\sigma_y}\right)$	$M_2 = \frac{\rho}{\sigma_y}$



Each bubble depends only on material properties; its position does not depend on the values of S^*, L^*, or F_f^*

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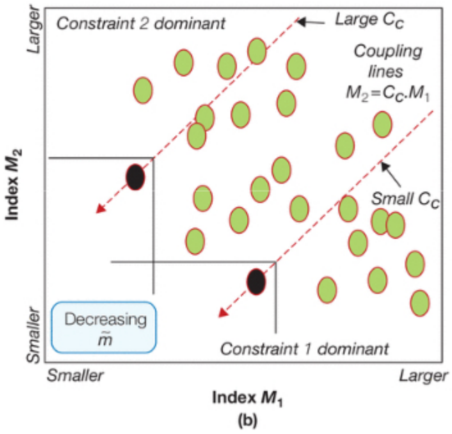
The Graphical Method



Performance equation	Material index
$m_1 = L^{*2}S^{*}\left(\frac{\rho}{E}\right)$ ⇨	$M_1 = \frac{\rho}{E}$
$m_2 = L^{*}F^{*}_{f}\left(\frac{\rho}{\sigma_y}\right)$ ⇨	$M_2 = \frac{\rho}{\sigma_y}$

IF m1=m2 THAN

$$M_2 = \left(\frac{L^* S^*}{F_f^*}\right) M_1$$

This describes a line of slope 1, in a position that depends on the value of LS/Ff. We refer to this line as the coupling line, and to LS/F_f as the coupling constant, symbol C_c.

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Graphical Solution:

$m_1 = L^2 S\left(\frac{\rho}{E}\right)$

$m_2 = LF\left(\frac{\rho}{\sigma_y}\right)$

This is what we know

M1 = $\left(\frac{\rho}{E}\right)$

M2 = $\left(\frac{\rho}{\sigma_y}\right)$

make $m_1 = m_2$

Solve for M2 $M2 = \frac{LS}{F} M1$ $\frac{L^* S^*}{F^*} = coupling\ \text{constant}/factor, C_C$

$$\log(M2) = Log(M1) + \log\left(\frac{L^* S^*}{F^*}\right)$$

Straight line, slope = 1
y-intcpt = LS/F

On logarithmic scales

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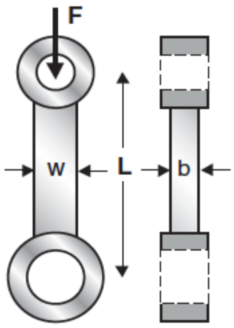
Case Study # 1: Con-Rods for High-Performance Engines

Design Goal: lighter, stronger con-rods for high performance engines



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The problem:

A connecting rod in a high-performance engine, compressor, or pump is a critical component: if it fails, catastrophe follows. Yet to minimize inertial forces and bearing loads it must weigh as little as possible, implying the use of **light**, **strong materials**, stressed near their limits. .

When minimizing cost is the objective, con-rods are frequently made of cast iron because it is so cheap

What are the best materials for con-rods when the objective is to **maximize performance**?

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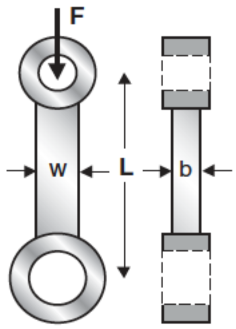
The design requirements: connecting rods

Function	Connecting rod for reciprocating engine or pump
Constraints	• Must not fail by high-cycle fatigue, or • Must not fail by elastic buckling • Stroke, and thus con-rod length *L*, specified
Objective	Minimize mass
Free variables	• Cross-section A • Choice of material



For simplicity assume that the shaft has a rectangular section

$$A = b \times w$$

The objective function is an equation for the mass that we approximate as;

$$m = \beta A L \rho$$

L = length of the con-rod

ρ = density of the material

β = a constant multiplier to allow for the mass of the bearing housings

A = the cross-section of the shaft

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The objective function is an equation for the mass that we approximate as;

$$m = \beta A L \rho$$

L = length of the con-rod

ρ = density of the material

β = a constant multiplier to allow for the mass of the bearing housings

A = the cross-section of the shaft

The fatigue constraint requires that; $\frac{F}{A} \leq \sigma_e$ — σ_E: endurance limit of the material used for con-rod

Material Index 1 → $m_1 = \beta F L \left(\frac{\rho}{\sigma_e}\right)$

The buckling constraint requires that the peak compressive load F does not exceed the Euler buckling load:

$$F \leq \frac{\pi^2 E I}{L^2}$$

Check Appendix A for the value I **($I = b^3 w/12$)**

$b = \alpha w$ α = dimensionless "shape constant"

Material Index 2 → $m_2 = \beta \left(\frac{12F}{\alpha \pi^2}\right)^{1/2} L^2 \left(\frac{\rho}{E^{1/2}}\right)$

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If the specifications are;

L = 200 mm, F = 50 kN α =0.8, β = 1.5

Material	ρ (kg/m^3)	E (GPa)	σ_e (MPa)	m_1 (kg)	m_2 (kg)	$\tilde{m} = max\ (m_1, m_2)$ (kg)
Nodular cast iron	7150	178	250	0.43	0.22	0.43
HSLA steel 4140 (o.q. T-315)	7850	210	590	0.20	0.28	0.28
Al S355.0 casting alloy	2700	70	95	0.39	0.14	0.39
Duralcan Al–SiC(p) composite	2880	110	230	0.18	0.12	0.18
Titanium 6Al 4V	4400	115	530	0.12	0.17	0.17

The table lists the mass **m**1 of a rod that will just meet the fatigue constraint, and the mass **m**2 that will just meet that on buckling

The quantity ≈m in the last column of the table is the larger of m1 and m2 for each material; it is the lowest mass that meets both constraints.

1. Titanium alloy Ti 6Al 4V.
2. Duralcan 6061–20% SiC

} Both weigh less than half as much as a cast-iron rod

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RESTRICTIONS

(1) This Method assumes some ‘‘pre-selection’’ procedure has been used to obtain the materials listed in the table, but does not explain how this is to be done.

(2) The results apply only to the values of F and L listed above change these, and the selection changes.

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The mass of the rod that will survive both fatigue and buckling is the larger of the two masses **m**1 and **m**2

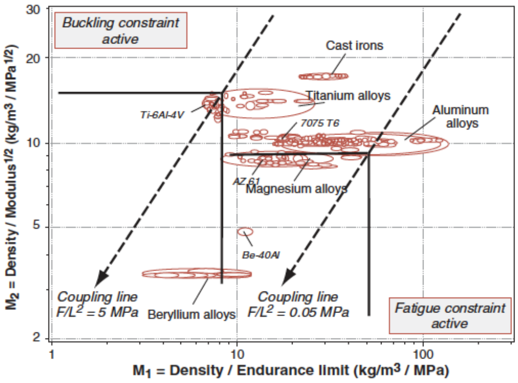
$$m_1 = \beta F L \left(\frac{\rho}{\sigma_e}\right) \quad \text{m1 = m2} \quad m_2 = \beta\left(\frac{12F}{\alpha\pi^2}\right)^{1/2} L^2\left(\frac{\rho}{E^{1/2}}\right)$$

$$M_2 = \left[\left(\frac{\alpha\pi^2}{12}\cdot\frac{F}{L^2}\right)^{1/2}\right]\cdot M_1$$

Coupling Constant (C)

Coupling lines for two values of F/L^2 are plotted on it, taking $\alpha = 0.8$

Materials with the optimum combination of **M**1 and **M**2 are identified by creating a chart with these indices as axes



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Materials for high-performance con-rods

Material	Comment
Magnesium alloys	AZ61 and related alloys offer good all-round performance
Titanium alloys	Ti-6-4 is the best choice for high F/L^2
Beryllium alloys	The ultimate choice, but difficult to process and very expensive
Aluminum alloys	Cheaper than titanium or magnesium, but lower performance

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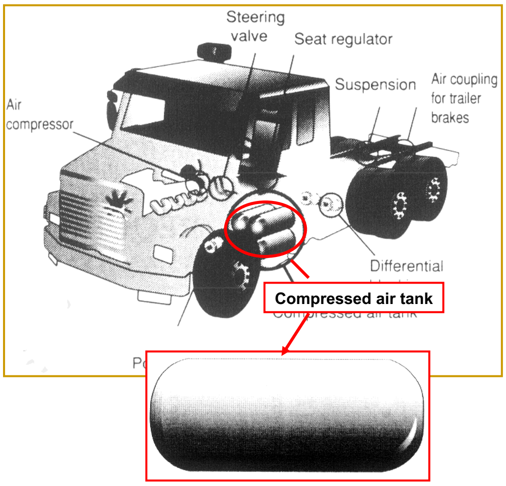
Case Study # 2: Air Cylinder for a Truck

Design Goal: lighter, safe air cylinders for trucks



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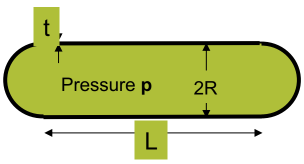
Case study: Air cylinder for truck



Density ρ
Yield strength σ_y
Fracture toughness K_{1c}

Function	*Pressure vessel*
Objective	***Minimise mass***
Constraints	*Dimensions L, R, pressure p, given* **Safety: *must not fail by yielding*** **Safety: *must not fail by fast fracture*** *Must not corrode in water or oil* *Working temperature -50 to +100^{0}C*
Free variables	*Wall thickness, t; choice of material*

Conflicting constraints lead to competing performance metrics

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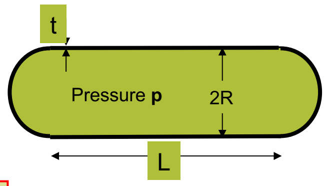
Air cylinder for truck



Density ρ
Yield strength σ_y
Fracture toughness K_{1c}

What is the free variable?

Objective: mass

Vol of material in cylinder wall

Aspect ratio, α

$$m = (2\pi R t L + 4\pi R^2 t)\rho = 2\pi R t L\, \rho \left(1 + \frac{2R}{L}\right)$$

Stress in cylinder wall

$$\sigma = \frac{pR}{2t} < \frac{\sigma_f}{S}$$

Failure stress

Safety factor

May be either σ_y or σ_f

Eliminate t

transpose

$$m = 2\pi R^2 L \alpha p S \left[\frac{\rho}{\sigma_f}\right]$$

$$m^* = \frac{m}{2\pi R^2 L \alpha p S} = \left[\frac{\rho}{\sigma_f}\right]$$

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Air cylinder : graphical solution using CES charts

CES Stage 1; apply simple (non conflicting) constraints:
working temp up to 100^{0}C, resist organic solvents etc.

CES Stage 2: evaluate conflicting performance metrics:

Must not yield: $\sigma_{f1} = \sigma_y$

Must not fracture $\sigma_{f2} = \frac{K_{1c}}{\sqrt{\pi a}}$

S = safety factor
a = crack length
σ_y = yield strength
K_{1c} = Fracture toughness

$$m_1^* = \frac{\rho}{\sigma_y}$$

$$m_2^* = \frac{\rho}{K_{1c}/\sqrt{\pi a}}$$

Competing performance metrics for minimum mass

Rank by the more restrictive of the two

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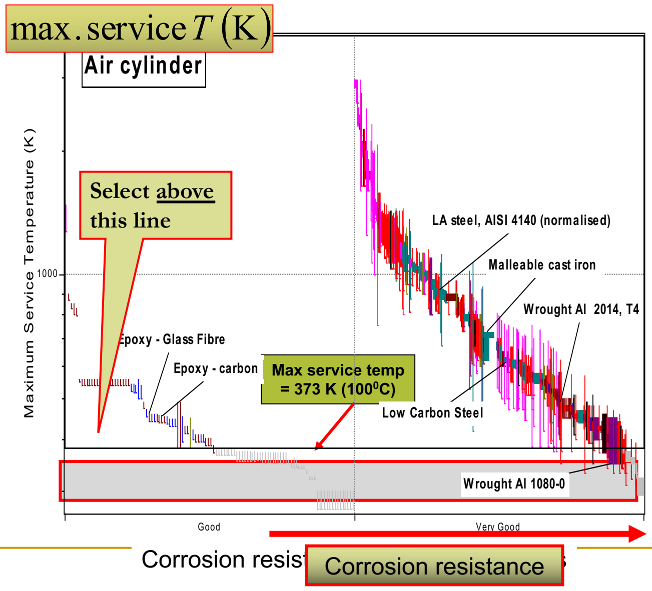
Air cylinder - Simple (non- conflicting) constraints



CES Stage 1:
- Impose constraints on corrosion in organic solvents
- Impose constraint on maximum working temperature

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Summary

- Real designs are **over-constrained** and many have **multiple objectives**
- Method of **maximum restrictiveness** copes with **conflicting multiple constraints**
- Analytical method useful but depends on the particular conditions set and lacks the visual power of the graphical method
- Graphical method produces a more general solution

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